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INFERENCE FOR THE SUJATHA DISTRIBUTION UNDER ADAPTIVE PROGRESSIVE CENSORING WITH APPLICATIONS TO GLASS FIBRE STRENGTH AND BALL BEARING FATIGUE LIFE

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ABSTRACT

The paper proposes a comprehensive inferential procedure for the Sujatha distribution under an adaptive Type-II progressive hybrid censoring scheme. The Sujatha distribution is a recently introduced one-parameter lifetime distribution with greater flexibility than the traditional exponential and Lindley distributions. The adaptive censoring design incorporates a threshold time T^* allowing for the censoring removal process to be controlled during the experiment. Maximum likelihood estimates of the parameter of interest are obtained using the Newton–Raphson method through the maxLik package in R. Furthermore, asymptotic confidence intervals are obtained using the normal approximation method and the log transformation method. Bayesian inference is carried out using a Markov Chain Monte Carlo method based on the Metropolis–Hasting's algorithm with two different gamma prior distributions for the parameter of interest: one non-informative "Gamma" (0.001,0.001) prior and another informative "Gamma" (2,2) prior. Parametric bootstrap confidence intervals using the percentile method and the bootstrap-t method are also provided for inference purposes. A detailed simulation study based on six different plans with different sample sizes and different proportions of censoring is carried out to show the consistency of the estimators and the shrinking of the interval estimates with an increase in sample size. The proposed inferential methods are applied to two real data sets: one for glass fibre strength data and another for ball bearing fatigue life data. Goodness-of-fit tests such as the Kolmogorov–Smirnov test, Anderson–Darling test, and Cramér–von Mises test are carried out for checking the appropriateness of the Sujatha distribution for the data sets considered in this paper. In addition, AIC, BIC, AICc, HQIC, CAIC, and DIC values are computed for choosing between different distributions for the data sets considered in this paper.

KEYWORDS: Sujatha Distribution; Adaptive Type-II Progressive Hybrid Censoring; Maximum Likelihood Estimation; Bayesian Inference; MCMC; Survival Analysis; Reliability; Hazard Function; Simulation Study; Bootstrap Confidence Intervals.

MSC Codes: 62N01; 62N02; 62F10; 62F15; 62N05; 62P30.

1. INTRODUCTION

The lifetime data analysis is considered a central theme in reliability, biomedical studies, and actuarial studies. In most practical problems, it is not always possible or cost-effective to follow all units until they fail, which has led to the use of censoring schemes that allow experiments to terminate before all units have failed [21, 24].

The progressive censoring is an extension of Type-II censoring, which allows for the removal of some units at intermediate failure times, thus providing a more flexible approach to experimentation [3, 4]. In Type-II progressive hybrid censoring, a time constraint T is also incorporated, which allows for termination of the experiment at a specified time T or at the m-th failure, whichever is earlier [19]. This approach, however, may result in a large number of removals not being executed, which may reduce the precision of the estimator.

To overcome this difficulty, Ng et al. [29] suggested an adaptive Type-II progressive censoring scheme, which allows for an adjustment in the removal pattern at a pre-specified threshold time T^*. When the m-th failure does not occur until after time T^*, all removals after that time are set to zero, which means that subsequent failures are not removed, thus ensuring that exactly m failures are observed. This approach was used to analyze different lifetime distributions, including inverse Weibull distribution [42], dependent competing risk models [6], and the inverted exponentiated Rayleigh distribution [43]. A comprehensive review of different censoring schemes is given in Oommen et al. [31] and Zhuang et al. [44]. Some of the recent developments in progressive censoring include unified approaches [7], determination of optimal sample sizes [12], and generalized power models [1].

The Sujatha distribution was introduced by Shanker [37] as a one-parameter lifetime model with probability density function

$$f(x;\theta)=\frac{\theta^3}{\theta^2+\theta+2}(1+x+x^2)e^{-\theta x},\quad x>0,\ \theta>0.$$

The above distribution is a mixture of exponential, gamma, and generalized gamma distributions; thus, it provides a more flexible distribution compared to exponential or Lindley [23, 11] distributions but remains a one-parameter distribution. Various extensions of Sujatha distributions have been studied, such as exponentiated [2, 32], size-biased [38], two-parameter [27, 26, 28], and generalized [39] variants, as well as Poisson–Sujatha models for count data [30]. Related one-parameter distributions include the Shanker [35] and Aradhana [36] distributions.

Despite the increasing interest in adaptive progressive censoring plans and Sujatha distributions, no attempt has been made thus far to integrate the two concepts in a systematic manner. In this article, we attempt to bridge this gap in the literature by providing a comprehensive inferential procedure for Sujatha distributions based on the adaptive Type-II progressive hybrid censoring scheme. Specifically, we propose the following inferential tools for the Sujatha distribution based on the adaptive Type-II progressive hybrid censoring scheme: derivation of the likelihood function for Sujatha distributions based on the adaptive Type-II progressive hybrid censoring scheme; maximum likelihood estimation of the parameter of Sujatha distributions using the Newton–Raphson algorithm with profile likelihood inference and two different methods of asymptotic confidence intervals; Bayesian inference of Sujatha distributions using the Metropolis–Hastings MCMC algorithm with two different gamma prior distributions along with Bayesian credible intervals and highest posterior density intervals; parametric bootstrap confidence intervals using percentile and bootstrap-t approaches; a comprehensive simulation study based on six different experimental designs for the assessment of the performance of the proposed inferential methods in terms of RMSE, MAB, CP, and AIL; and real data analysis with goodness-of-fit testing and model selection.

The rest of this article is organized as follows. Section 2 introduces the Sujatha distribution and the adaptive censoring scheme. Section 3 develops the maximum likelihood method for estimating the parameter of Sujatha distributions based on the adaptive Type-II progressive hybrid censoring scheme. Section 4 develops the Bayesian method of inference using the MCMC algorithm with two different prior distributions. Section 5 develops parametric bootstrap confidence intervals. Section 6 provides a comprehensive simulation study. Section 7 provides real data analysis. Section 8 concludes.

2. THE MODEL

The Sujatha distribution with parameter $\theta>0$ has the probability density function given in (1). The corresponding cumulative distribution function is

$$F(x;\theta)=1-\left(1+\frac{\theta x(\theta x+\theta+2)}{\theta^2+\theta+2}\right)e^{-\theta x},\quad x>0.$$

The survival function is

$$S(x;\theta)=1-F(x;\theta)=\left(1+\frac{\theta x(\theta x+\theta+2)}{\theta^2+\theta+2}\right)e^{-\theta x},$$

and the hazard rate function is

$$h(x;\theta) = \frac{f(x;\theta)}{S(x;\theta)} = \frac{\theta^3(1+x+x^2)}{\theta^2+\theta+2+\theta x(\theta x+\theta+2)}.$$

The hazard rate increases for all values of $\theta > 0$, and therefore, the Sujatha distribution is applicable for modelling aging phenomena and wear-out [37]. The respective distribution functions are given in Figure 1.

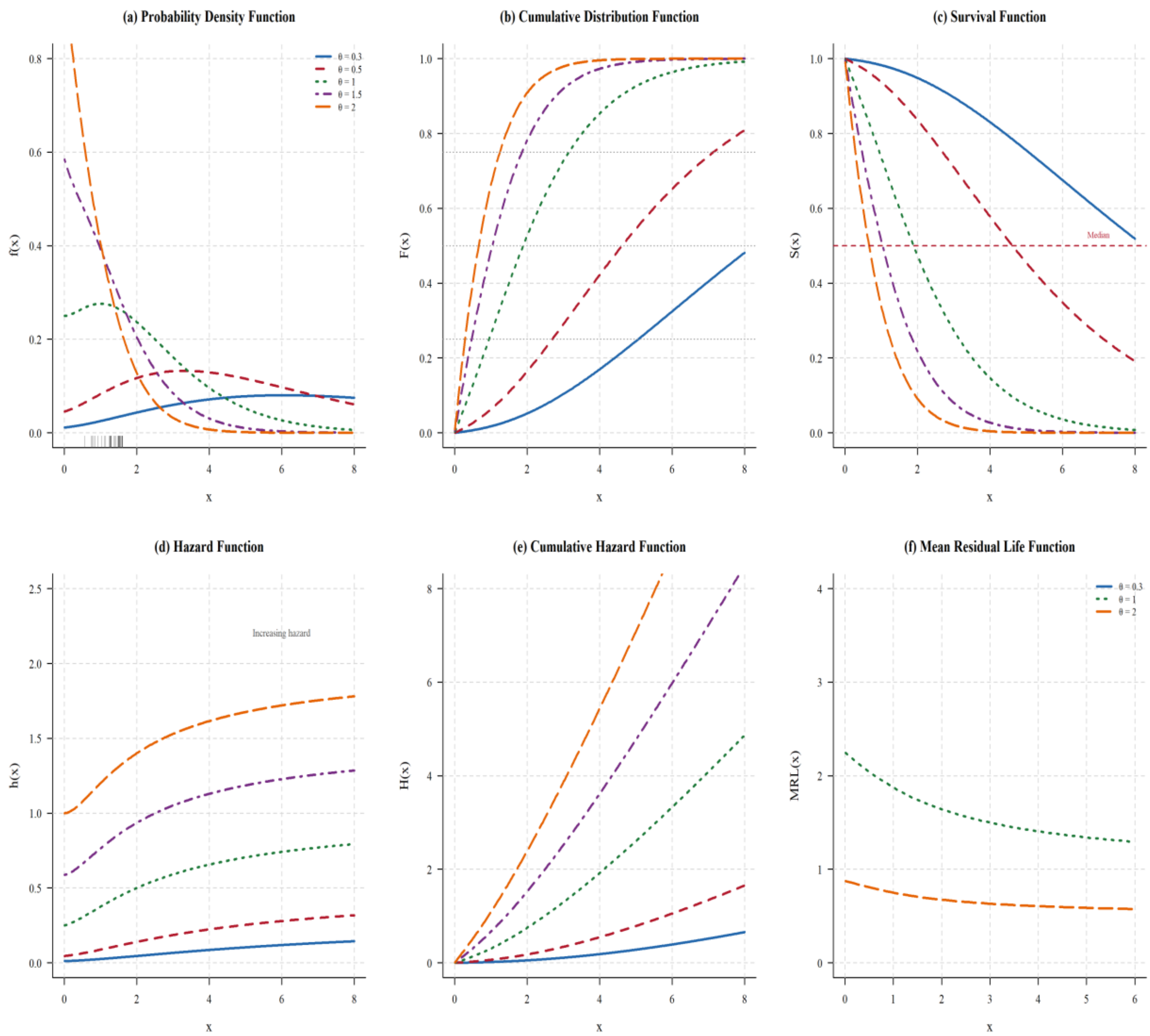

Figure 1: Probability Density, Cumulative Distribution, Survival, And Hazard Rate Functions of the Sujatha Distribution for Selected Values Of.

For example, in a life test where n identical items are put on test simultaneously, a progressive censoring scheme is defined by the set of integers $(R_1, R_2, \dots, R_m)$, which satisfy the relation $\sum_{i=1}^{m} R_i = n - m$. In the adaptive Type-II progressive hybrid censoring scheme [29], a threshold time T^* is pre-specified. Let $x_{1:m:n} < x_{2:m:n} < \cdots < x_{m:m:n}$ denote the ordered failure times and define the adaptive index

$$J = \max\{j : x_{j:m:n} \le T^*\}.$$

The removal scheme is then adjusted as follows:

$$R_i^* = \begin{cases} R_i, & i = 1, \dots, J, \\ 0, & i = J+1, \dots, m-1, \\ n - m - \sum_{i=1}^{J} R_i, & i = m. \end{cases}$$

This ensures that exactly m failures are always observed, regardless of when the threshold time is reached.

Given the adaptive Type-II progressively hybrid censored sample $\mathbf{x} = (x_{1:m:n}, \dots, x_{m:m:n})$ with adjusted removal scheme $(R_1^*, \dots, R_m^*)$, the likelihood function is

$$L(\theta \mid \mathbf{x}) = C \prod_{i=1}^{m} f(x_i;\theta) \prod_{i=1}^{m} [S(x_i;\theta)]^{R_i^*},$$

where C is a constant that does not depend on θ. The log-likelihood function is

$$\ell(\theta) = \text{const} + \sum_{i=1}^{m} \log f(x_i;\theta) + \sum_{i=1}^{m} R_i^* \log S(x_i;\theta).$$

3 MAXIMUM LIKELIHOOD ESTIMATION

Plugging in the density of the Sujatha distribution given in equation (1) and the corresponding survival function given in equation (3) into the log-likelihood given in equation (8), we can then solve the score function $U(\theta) = \partial\ell/\partial\theta = 0$ numerically with the help of a Newton–Raphson algorithm, which is provided in an R package named maxLik [16]. The observed Fisher information $I(\hat{\theta}) = -\partial^2\ell/\partial\theta^2|_{\theta=\hat{\theta}}$ is used to obtain the asymptotic variance of $\hat{\theta}$. The detailed algorithmic steps for maximum likelihood estimation are presented in Algorithm 1.

Algorithm 1 Maximum Likelihood Estimation under Adaptive Type-II Progressive Hybrid Censoring

Require: Ordered failure times $x_{1:m:n} < \cdots < x_{m:m:n}$, removal scheme $\mathbf{R} = (R_1, \ldots, R_m)$, threshold time T^*, initial value $\theta^{(0)}$, tolerance $\epsilon = 10^{-8}$

Ensure: MLE $\hat{\theta}$, $\hat{R}(x)$, $\hat{h}(x)$ with standard errors and confidence intervals

1. Determine adaptive index $J = \max\{j: x_{j:m:n} \leq T^*\}$
2. Adjust removal scheme: set $R_i^* = 0$ for $i > J$ and redistribute remaining units to R_J^*
3. Construct the log-likelihood: $\ell(\theta) = \sum_{i=1}^{m} \log f(x_i;\theta) + \sum_{i=1}^{m} R_i^* \log S(x_i;\theta)$
4. where $f(x;\theta) = \frac{\theta^3}{\theta^2+\theta+2}(1 + x + x^2)e^{-\theta x}$ and $S(x;\theta) = \left(1 + \frac{\theta x(\theta x+\theta+2)}{\theta^2+\theta+2}\right)e^{-\theta x}$
5. Compute score function $U(\theta) = \partial\ell/\partial\theta$
6. Compute observed Fisher information $I(\theta) = -\partial^2\ell/\partial\theta^2$
7. **repeat**
   $\theta^{(k+1)} \leftarrow \theta^{(k)} + \left[I(\theta^{(k)})\right]^{-1} U(\theta^{(k)})$ {Newton–Raphson via maxLik}
   until $|\theta^{(k+1)} - \theta^{(k)}| < \epsilon$
8. $\hat{\theta} \leftarrow \theta^{(k+1)}$; compute $\hat{R}(x) = S(x;\hat{\theta})$ and $\hat{h}(x) = f(x;\hat{\theta})/S(x;\hat{\theta})$
9. Compute $\widehat{\mathrm{Var}}(\hat{\theta}) = \left[I(\hat{\theta})\right]^{-1}$; apply delta method for SE of $\hat{R}(x)$ and $\hat{h}(x)$
10. Construct ACI-NA: $\hat{\theta} \pm z_{\alpha/2}\,\mathrm{SE}(\hat{\theta})$; ACI-NL via log-transform
11. **return** $\hat{\theta}$, $\hat{R}(x)$, $\hat{h}(x)$, SEs, ACI-NA, ACI-NL, AIC, BIC, AICc, HQIC

By the asymptotic normality of the MLE, $\hat{\theta} \dot{\sim} N\left(\theta, \left[I(\hat{\theta})\right]^{-1}\right)$, and a $100(1-\alpha)\%$ confidence interval based on the normal approximation (ACI-NA) is

$$\hat{\theta} \pm z_{\alpha/2}\mathrm{SE}(\hat{\theta}),$$

where $\mathrm{SE}(\hat{\theta}) = \left[I(\hat{\theta})\right]^{-1/2}$. Given that $\theta > 0$, a log-transformed confidence interval (ACI-NL) that has better small sample coverage properties can be constructed using the delta method for $\log\hat{\theta}$ [9]:

$$\hat{\theta}\exp\left(\pm z_{\alpha/2}\frac{\mathrm{SE}(\hat{\theta})}{\hat{\theta}}\right).$$

Confidence intervals for $R(x)$ and $h(x)$ are derived using the delta method [17]. Profile likelihood analysis provides a visual assessment of estimation precision and likelihood shape [9, 5].

4 BAYESIAN ESTIMATION

A conjugate-style gamma prior is placed on θ:

$$\pi(\theta) = \frac{b^a}{\Gamma(a)}\theta^{a-1}e^{-b\theta}, \quad \theta > 0.$$

Two specifications are considered: Prior 1, Gamma(0.001,0.001), which is essentially non-informative and lets the data dominate, and Prior 2, Gamma(2,2), which incorporates mild prior information centred at $\theta = 1$ [10]. The posterior distribution is

$$\pi(\theta \mid \mathbf{x}) \propto L(\theta \mid \mathbf{x})\pi(\theta),$$

which does not admit a closed-form expression and is sampled using MCMC.

A Metropolis–Hasting's algorithm with a log-normal random walk is used for this purpose [25, 15, 34]. The values for the new θ^* are generated using $\theta^* = \exp\left(\log\theta^{(t-1)} + \epsilon\right)$, where $\epsilon \sim N(0, \sigma_p^2)$. The acceptance probability for this algorithm includes the Jacobian of the log-transformation. The value of σ_p is varied during the burn-in phase of the algorithm such that an acceptance rate of approximately 0.44 is achieved for optimal convergence of the algorithm for univariate distributions. The algorithm is run for 15,000 iterations with a burn-in of 5,000 and a thinning factor of 5, giving us a total of 2,000 posterior samples from the posterior distribution of θ. Posterior summary statistics such as posterior means, medians, standard deviations, Bayesian credible intervals (BCI) at 95%, and highest posterior density intervals (HPD) at 95% are then obtained from these posterior samples of θ. For derived quantities $R(x)$ and $h(x)$, posterior samples are generated by evaluating the survival and hazard functions at each posterior draw of θ. The complete MCMC algorithm is given in Algorithm 2. The deviance information criterion (DIC) [10, 18] is used for Bayesian model selection.

Algorithm 2 Bayesian Estimation via Metropolis–Hastings MCMC

Require: Data $(x_1, \ldots, x_m)$, removal scheme $\mathbf{R}$, prior $\pi(\theta) = \mathrm{Gamma}(a, b)$ with $(a, b) \in \{(0.001, 0.001), (2,2)\}$, MCMC settings: $N = 15{,}000$, burn-in = 5,000, thin = 5

Ensure: Posterior mean, median, SD; 95% BCI; 95% HPD; DIC

1. Initialize $\theta^{(0)} = \hat{\theta}_{\mathrm{MLE}}$; set proposal SD $\sigma_p =$

$0.15 \cdot \theta^{(0)}$

2. **for** $t = 1$ **to** N **do**
   a. Propose $\theta^* = \exp(\log\theta^{(t-1)} + \epsilon)$, $\epsilon \sim N(0, \sigma_p^2)$ {Log-normal random walk}
   b. Compute log-posterior: $\log\pi(\theta \mid \mathbf{x}) = \ell(\theta) + \log\pi(\theta)$
   c. Acceptance probability: $\alpha = \min\left(1, \frac{\pi(\theta^*|\mathbf{x})}{\pi(\theta^{(t-1)}|\mathbf{x})} \cdot \frac{\theta^*}{\theta^{(t-1)}}\right)$
   d. Draw $u \sim \text{Uniform}(0,1)$
   e. **if** $u < \alpha$ **then** $\theta^{(t)} \leftarrow \theta^*$ **else** $\theta^{(t)} \leftarrow \theta^{(t-1)}$ **end if**
   f. Adapt σ_p during burn-in to target acceptance rate ≈ 0.44
   end for
3. Discard first 5000 iterations; apply thinning factor 5 to obtain $\{\theta^{(j)}\}_{j=1}^M$
4. Compute posterior summaries: $\hat{\theta}_{\text{Bayes}} = M^{-1}\sum\theta^{(j)}$, $\hat{R}(x) = M^{-1}\sum S(x; \theta^{(j)})$, $\hat{h}(x) = M^{-1}\sum h(x; \theta^{(j)})$
5. Evaluate convergence: ESS, Geweke test, trace plots
6. **return** Posterior mean, median, SD, 95% BCI, 95% HPD, DIC

5 BOOTSTRAP CONFIDENCE INTERVALS

Parametric bootstrap procedures can be used as an alternative to interval estimation that does not rely on asymptotic normality [8, 13, 20]. In the parametric bootstrap approach, $B = 500$ bootstrap data sets are simulated from the Sujatha model, and then the adaptive censoring scheme is used to estimate the model. Two types of intervals will be used: the percentile interval method and the bootstrap-t interval method. The bootstrap approach is given in Algorithm 3.

Algorithm 3 Parametric Bootstrap Confidence Intervals

Require: Original MLE $\hat{\theta}$, sample configuration $(n, m, \mathbf{R})$, threshold time T^*, number of bootstrap replications $B = 500$

Ensure: Bootstrap SE, percentile CI, bootstrap-t CI for θ, $R(x)$, $h(x)$

1. **for** $b = 1$ **to** B **do**
   a. Generate n observations from Sujatha$(\hat{\theta})$
   b. Sort to obtain order statistics $Y_{1:n} < \cdots < Y_{n:n}$
   c. Apply adaptive Type-II progressive hybrid censoring:
   - Determine $J^{*(b)} = \max\{j: Y_{j:m:n} \le T^*\}$; adjust removal scheme
   - Retain first m failures with adaptive removals

   d. Compute bootstrap MLE $\hat{\theta}^{*(b)}$ via Algorithm 1
   e. Compute $\hat{R}^{*(b)}(x) = S(x; \hat{\theta}^{*(b)})$ and $\hat{h}^{*(b)}(x) = h(x; \hat{\theta}^{*(b)})$
   end for
2. Compute bootstrap SE: $\widehat{\text{SE}}_{\text{boot}}(\hat{\theta}) = \sqrt{(B-1)^{-1}\sum_{b=1}^{B}\left(\hat{\theta}^{*(b)} - \bar{\theta}^*\right)^2}$
3. Percentile CI: $\left(\hat{\theta}^*_{(\alpha/2)}, \hat{\theta}^*_{(1-\alpha/2)}\right)$
4. Bootstrap-t CI: $\hat{\theta} \pm t_{\alpha/2, B-1} \cdot \widehat{\text{SE}}_{\text{boot}}$
5. **return** Bootstrap SE, percentile CI, bootstrap-t CI for θ, $R(x)$, $h(x)$

6 SIMULATION STUDY

In order to evaluate the finite sample properties of the proposed methods, a Monte Carlo simulation study is carried out. For the simulation, the true parameter is chosen to be $\theta = 1.0$, and the associated true values are $R(1.89) = 0.5000$ and $h(1.89) = 0.4881$. Six simulation plans are considered, and they are divided into two groups. Group I considers variations in the sample size, where the sample sizes are chosen to be $n = 20,30,50$, with moderate censoring. Group II considers variations in the censoring level, where the censoring is chosen to be 10%, 25%, and 40%, and the sample size is fixed at $n = 30$. For all the simulation plans, the number of Monte Carlo replications is chosen to be $M = 200$, and the conventional removal scheme is employed, where all the removals are made in the last observed failure, i.e., $R_m = n - m$. For all the simulation plans, the threshold time T^* is chosen to be the median of the simulated data, which is a truly adaptive censoring mechanism. All the simulation plans are summarized in Table 1.

Table 1: Simulation Study Design and Configuration.

Plan	Description	□	□	Cens. Rate	□	Removal Scheme
I.1	Varying □	20	15	25%	200	□□ = □ − □
I.2	Varying □	30	22	27%	200	□□ = □ − □
I.3	Varying □	50	35	30%	200	□□ = □ − □
II.1	Varying cens.	30	27	10%	200	□□ = □ − □
II.2	Varying cens.	30	22	25%	200	□□ = □ − □
II.3	Varying cens.	30	18	40%	200	□□ = □ − □

True parameter: $\theta = 1.0$; Monte Carlo replications: $M = 200$. Estimation methods: MLE, Bayesian (Prior 1: Gamma (0.001,0.001), Prior 2: Gamma (2,2)). Censoring: Adaptive Type-II

Progressive Hybrid with T^{*} = median of simulated data.

Four performance metrics are computed. The root means square error (RMSE) and mean absolute bias (MAB) measure point estimation accuracy:

$$\mathrm{RMSE}(\hat{\xi}) = \sqrt{\frac{1}{M}\sum_{k=1}^{M}\left(\hat{\xi}_k - \xi\right)^2}, \quad \mathrm{MAB}(\hat{\xi}) = \frac{1}{M}\sum_{k=1}^{M}\left|\hat{\xi}_k - \xi\right|,$$

where ξ stands for θ, $R(x)$, or $h(x)$. The coverage probability (CP) and average interval length (AIL) assess interval estimation performance.

Table 2 displays the values of the RMSE and MAB for all six plans and the estimation methods. Several patterns can be observed from the results. First, the values of the RMSE and MAB all decline as the sample size increases from $n = 20$ to $n = 50$, showing the consistency of all the estimators. Specifically, the values of the RMSE for the estimation of θ decline from 0.2495 (MLE, $n = 20$) to 0.1732 (MLE, $n = 50$), showing a reduction of approximately 31%. Second, the Bayesian estimator using Prior 2 always yields the minimum values of the RMSE and MAB for the estimation of θ and $h(x)$, showing the benefit of using the informative prior located near the true value. Third, the MLE performs as well as, or better than, the Bayesian estimators for the estimation of $R(x)$. This shows that the survival function is well-identified by the likelihood function.

Table 2: Simulation Study: RMSE And MAB For $\boldsymbol{\theta}$, $\boldsymbol{R(x)}$, And $\boldsymbol{h(x)}$.

RMSE

Plan	□	Cens.	MLE	Prior1	Prior2	MLE	Prior1	Prior2	MLE	Prior1	Prior2
			RMSE (□)			RMSE (□(□))			RMSE (□(□))		
□ = 20, □ = 15	20	25%	0.2495	0.2518	0.2358	0.0758	0.0770	0.0796	0.1274	0.1317	0.1186
□ = 30, □ = 22	30	27%	0.2119	0.2138	0.2052	0.0715	0.0721	0.0743	0.0986	0.1016	0.0948
□ = 50, □ = 35	50	30%	0.1732	0.1738	0.1700	0.0911	0.0919	0.0931	0.0709	0.0723	0.0696
10% cens	30	10%	0.1625	0.1631	0.1583	0.0489	0.0480	0.0484	0.0917	0.0937	0.0899
25% cens	30	27%	0.2035	0.2048	0.1975	0.0771	0.0779	0.0801	0.0913	0.0937	0.0881
40% cens	30	40%	0.1855	0.1859	0.1780	0.1667	0.1669	0.1685	0.0926	0.0921	0.0878

Mab

Plan	□	Cens.	MLE	Prior1	Prior2	MLE	Prior1	Prior2	MLE	Prior1	Prior2
			MAB (□)			MAB (□(□))			MAB (□(□))		
□ = 20, □ = 15	20	25%	0.1925	0.1939	0.1830	0.0584	0.0598	0.0624	0.0932	0.0963	0.0877
□ = 30, □ = 22	30	27%	0.1699	0.1714	0.1650	0.0593	0.0601	0.0624	0.0772	0.0794	0.0744
□ = 50, □ = 35	50	30%	0.1405	0.1410	0.1379	0.0815	0.0825	0.0837	0.0544	0.0557	0.0530
10% cens	30	10%	0.1246	0.1247	0.1220	0.0395	0.0388	0.0394	0.0703	0.0716	0.0692
25% cens	30	27%	0.1612	0.1621	0.1564	0.0640	0.0652	0.0677	0.0698	0.0712	0.0676
40% cens	30	40%	0.1373	0.1374	0.1321	0.1598	0.1601	0.1618	0.0686	0.0678	0.0652

True values: $\theta = 1.0$, $R(1.89) = 0.5000$, $h(1.89) = 0.4881$. Prior 1: Gamma (0.001,0.001); Prior 2: Gamma (2,2). RMSE: Root Mean Square Error; MAB: Mean Absolute Bias.

The simulation boxplots in Figure 2 provide a visual summary of the estimation variability across plans, illustrating the reduction in dispersion as sample size increases.

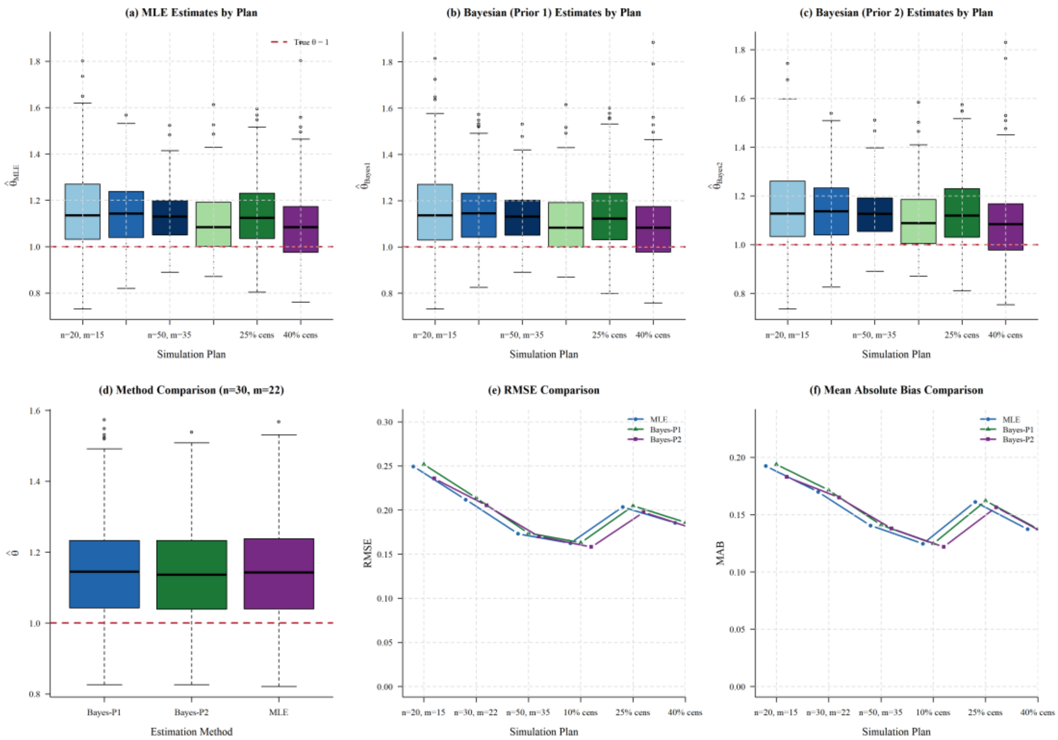

Figure 2. Boxplots Of MLE And Bayesian Estimates of Across the Six Simulation Plans.

Table 3 provides the coverage probabilities and average interval lengths for the four methods of interval estimation: ACI-NA, ACI-NL, BCI, and HPD. The coverage probabilities range from 0.760 to 0.935. The coverage probabilities for all methods are less than the nominal level of 95%, and this is not unexpected given the sample sizes and the finite MC replication number $M = 200$. The HPD intervals are all shorter than the BCI intervals because the former are the shortest credible intervals for each posterior draw. The average length of the intervals decreases with increasing sample size: 0.6710 for the ACI-NA with $n = 20$, and 0.4203 for the ACI-NA with $n = 50$. This result supports the idea that more information increases the precision of the intervals. For Group II, the 10% censoring plan has the highest coverage probabilities, up to 0.935 for the ACI-NA. This result supports the idea that less censoring improves the accuracy of the intervals.

Table 3: Simulation Study: Average Interval Length (AIL) And Coverage Probability (CP) For θ.

Plan	□	Cens.	ACI-NA	ACI-NL	BCI	HPD	ACI-NA	ACI-NL	BCI	HPD
			AIL				CP			
□ = 20, □ = 15	20	25%	0.6710	0.6804	0.6714	0.6549	0.880	0.810	0.830	0.830
□ = 30, □ = 22	30	27%	0.5422	0.5472	0.5423	0.5307	0.860	0.825	0.845	0.835
□ = 50, □ = 35	50	30%	0.4203	0.4228	0.4197	0.4112	0.825	0.760	0.785	0.805
10% cens	30	10%	0.4814	0.4853	0.4802	0.4691	0.935	0.875	0.900	0.905
25% cens	30	27%	0.5362	0.5412	0.5304	0.5200	0.865	0.815	0.830	0.855
40% cens	30	40%	0.5507	0.5565	0.5528	0.5404	0.930	0.915	0.920	0.915

ACI-NA: Asymptotic CI (Normal); ACI-NL: Asymptotic CI (Log-transform); BCI: Bayesian CI; HPD: Highest Posterior Density. Nominal coverage level: 95%.

The simulation trends across plans are illustrated in Figure 3, showing the decrease in RMSE and the behaviour of coverage probability as sample size and censoring proportion change.

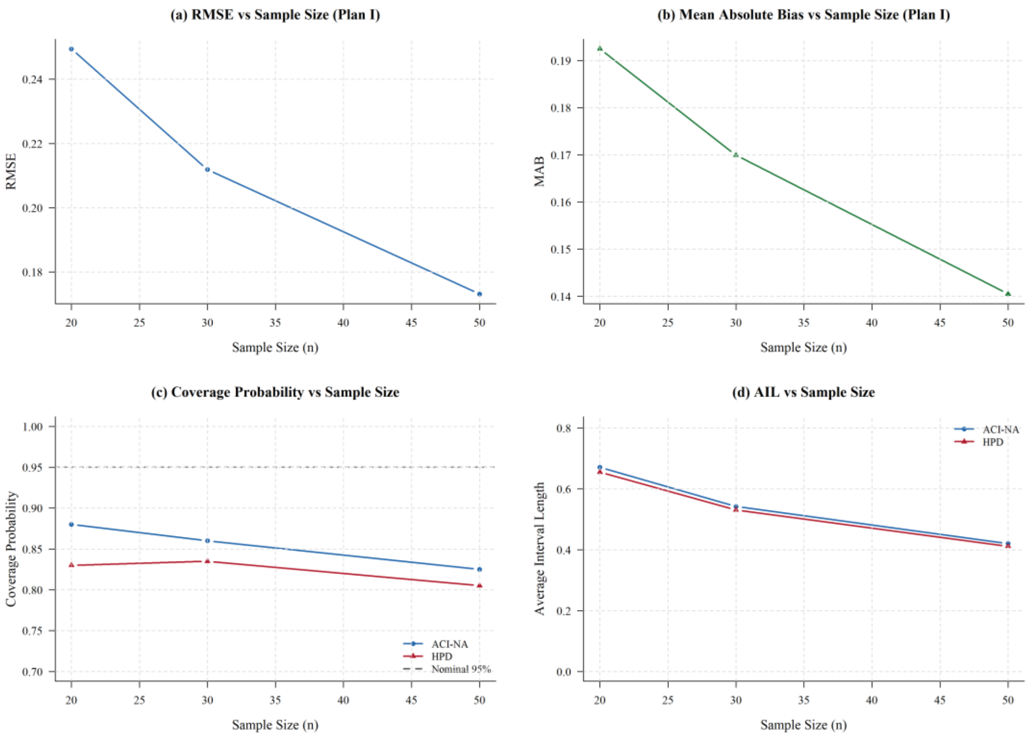

Figure 3. Trends In RMSE And Coverage Probability Across Simulation Plans for Varying Sample Sizes and Censoring Proportions.

7. APPLICATIONS

Two real datasets are analysed to illustrate the proposed methods. The combined data from both datasets are presented in Table 4, with summary statistics in Table 5.

Dataset 1: Glass Fibre Strength. This dataset contains $n = 63$ observations of the breaking strength of glass fibres, originally given by Smith and Naylor [41] and widely used in reliability analyses [40]. The progressive censoring scheme involves $m = 35$ observed failures, where the threshold time is given by $T^* = 1.5150$, and this results in an adaptive index of $J = 25$ and a 30% censoring rate.

Dataset 2: Ball Bearing Fatigue Life. The dataset contains $n = 23$ observations on the fatigue life of deep groove ball bearings in terms of millions of revolutions, as provided in Lieblein and Zelen [22]. The censoring scheme has $m = 18$ observed failures with a threshold time $T^* = 67.8000$, and this gives an adaptive index $J = 12$ with a censoring rate of 22%.

Table 4: Ordered Failure Times for Both Datasets Under Adaptive Type-II Progressive Hybrid Censoring.

Dataset 1: Glass Fiber (n = 63, m = 45)				Dataset 2: Ball Bearings (n = 23, m = 18)			
i	□□:□:□	□□	□̂(□□)	i	□□:□:□	□□	□̂(□□)
1	0.55	0	0.1968	1	17.88	0	0.0505
2	0.74	0	0.2633	2	28.92	0	0.1456
3	0.77	0	0.2737	3	33.00	0	0.1895
4	0.81	0	0.2875	4	41.52	0	0.2888
5	0.84	0	0.2979	5	42.12	0	0.2960
6	0.93	0	0.3285	6	45.60	0	0.3379
7	1.04	0	0.3654	7	48.48	0	0.3725
8	1.11	0	0.3884	8	51.84	0	0.4124
9	1.13	0	0.3949	9	51.96	0	0.4138
10	1.24	0	0.4302	10	54.12	0	0.4390
11	1.25	0	0.4334	11	55.56	0	0.4555
12	1.27	0	0.4397	12	67.80	5	0.5862
13	1.28	0	0.4428	13	68.64	0	0.5944
14	1.29	0	0.4459	14	68.64	0	0.5944
15	1.30	0	0.4490	15	68.88	0	0.5967
16	1.36	0	0.4675	16	84.12	0	0.7260
17	1.39	0	0.4767	17	93.12	0	0.7857

18	1.42	0	0.4857	18	98.64	0	0.8167
19	1.48	0	0.5035				
20	1.48	0	0.5035				
21	1.49	0	0.5064				
22	1.49	0	0.5064				
23	1.50	0	0.5094				
24	1.50	0	0.5094				
25	1.51	15	0.5123				
26	1.52	0	0.5152				
27	1.53	0	0.5181				
28	1.54	0	0.5209				
29	1.55	0	0.5238				
30	1.55	0	0.5238				
31	1.58	0	0.5324				
32	1.59	0	0.5352				
33	1.60	0	0.5380				
34	1.61	0	0.5408				
35	1.61	0	0.5408				

Table 5: Censoring Scheme Configuration and Descriptive Statistics.

Statistic	Glass Fiber Data	Ball Bearings
Censoring Configuration		
Total units (□)	50	23
Observed failures (□)	35	18
Censored units	15	5
Censoring rate	30.0%	21.7%
Threshold time □*	1.5150	67.8000
Adaptive index □	25	12
Descriptive Statistics		
Minimum	0.5500	17.8800
Maximum	1.6100	98.6400
Mean	1.3100	56.7133
Median	1.4200	53.0400
Std. Deviation	0.2920	21.5873
Skewness	-1.0042	0.2593
Kurtosis	2.8355	2.2500
Q1 (25%)	1.1850	42.9900
Q3 (75%)	1.5250	68.6400
IQR	0.3400	25.6500

Table 6 reports the maximum likelihood estimate and its standard error and confidence intervals for the two datasets. In the case of the glass-fiber dataset, the MLE of θ is $\hat{\theta} = 1.1912$ with standard error $= 0.1135$. It then leads to a survival estimate $\hat{R}(x) = 0.5143$ and a hazard estimate $\hat{h}(x) = 0.5827$ at the mission time. The asymptotic confidence interval under normal approximation for θ is $(0.9687, 1.4138)$, and the one under likelihood is $(0.9882, 1.4359)$. In the case of the ball-bearing dataset, the MLE is $\hat{\theta} = 0.0445$ with standard error $= 0.0058$. It then leads to a survival estimate $\hat{R}(x) = 0.5736$ and a hazard estimate $\hat{h}(x) = 0.0203$. From the profile likelihood plot for the two datasets, as shown in Figure 4, confirming that the log-likelihood is well-behaved and unimodal near the MLE.

Table 6: Maximum Likelihood Estimates with Various Confidence Intervals.

Dataset	Parameter	MLE	SE	ACI-NA	ACI-NL	Profile CI	Bootstrap CI
Dataset 1: Glass Fiber Data							
	□	1.1912	0.1135	(0.9687, 1.4138)	(0.9882, 1.4359)	–	(1.0952, 1.6488)
	□*(1.4)*	0.5143	0.0604	(0.3960, 0.6327)	–	–	(0.3081, 0.5667)
	□*(1.4)*	0.5827	0.0925	(0.4013, 0.7641)	–	–	(0.5056, 0.9751)
Dataset 2: Ball Bearings							
	□	0.0445	0.0058	(0.0331, 0.0559)	(0.0345, 0.0575)	–	(0.0381, 0.0649)
	□*(53.0)*	0.5736	0.0815	(0.4138, 0.7334)	–	–	(0.3247, 0.6652)
	□*(53.0)*	0.0203	0.0045	(0.0115, 0.0292)	–	–	(0.0155, 0.0373)

ACI-NA: Asymptotic CI (Normal Approximation); ACI-NL: Asymptotic CI (Log-transform)

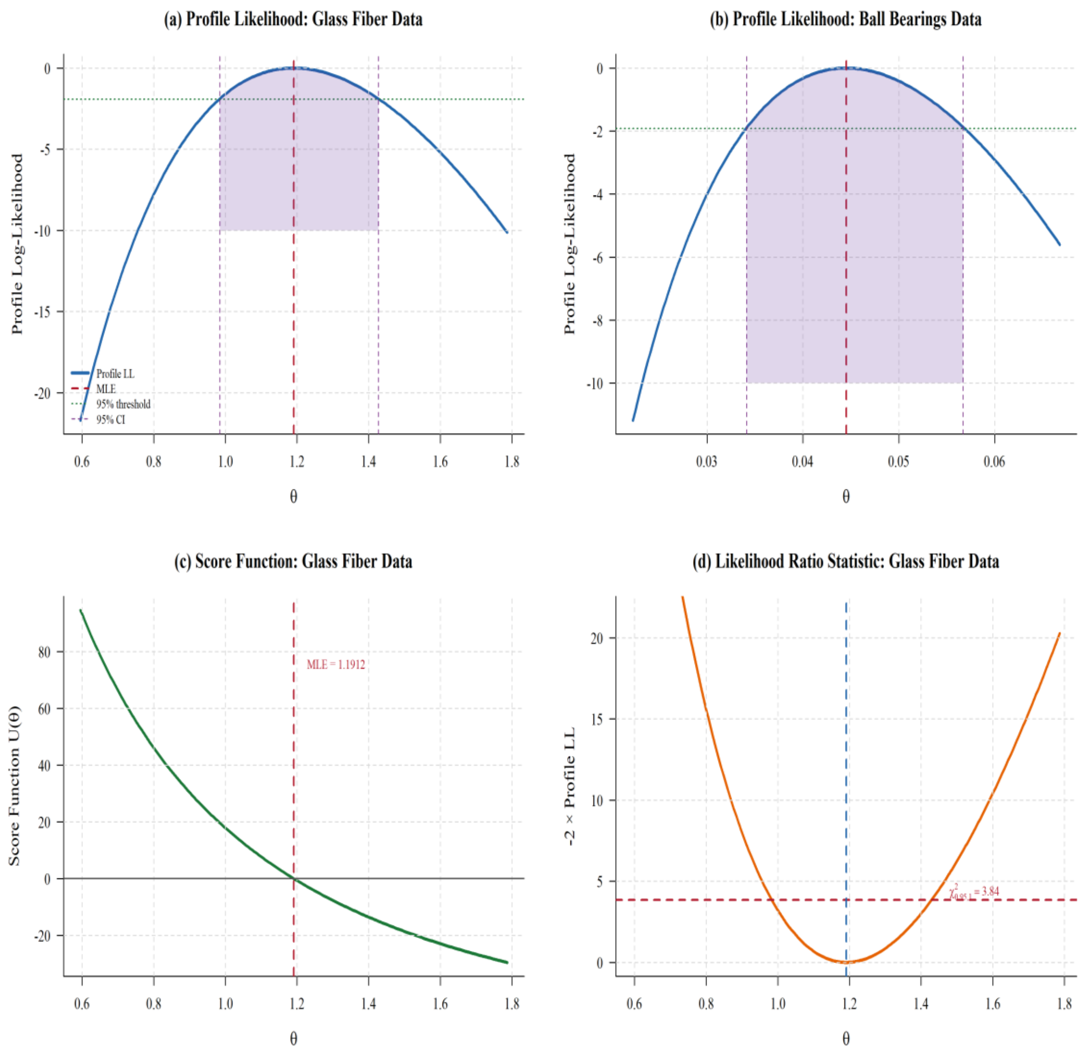

Figure 4: Profile Log-Likelihood Functions With 95% Confidence Bounds for the Glass Fibre and Ball Bearing Datasets.

For the assessment of convergence of the MCMC chains, the effective sample size (ESS), Geweke's diagnostic, trace plots, and autocorrelation plots were used. In addition, the results of the MCMC convergence diagnostics are provided in Table 7. In the case of the glass fiber data, the ESS varies between 1358 and 1849, with an acceptance rate between 0.418 and 0.472. In addition, three out of the four chains were able to satisfy Geweke's convergence test at a 5% significance level.

The chain that was not able to satisfy the Geweke convergence test was the chain "Glass Fiber," "Prior 1," with a Geweke $p = 0.0129$. However, this chain was able to achieve an ESS of 1849. The MCMC convergence diagnostic plots are provided in Figure 5.

Table 7: Mcmc Convergence Diagnostics Summary.

Dataset	Prior	Accept. Rate	ESS	Geweke Z	p-value	Converged
Glass Fiber	Prior 1	0.437	1848.6	2.486	0.0129	No
Glass Fiber	Prior 2	0.472	1644.9	-0.050	0.9602	Yes
Ball Bearings	Prior 1	0.418	1468.8	1.099	0.2717	Yes
Ball Bearings	Prior 2	0.419	1358.2	-0.716	0.4742	Yes

Prior 1: Gamma (0.001, 0.001); Prior 2: Gamma (2, 2). ESS: Effective Sample Size; Convergence: Geweke p-value > 0.05.

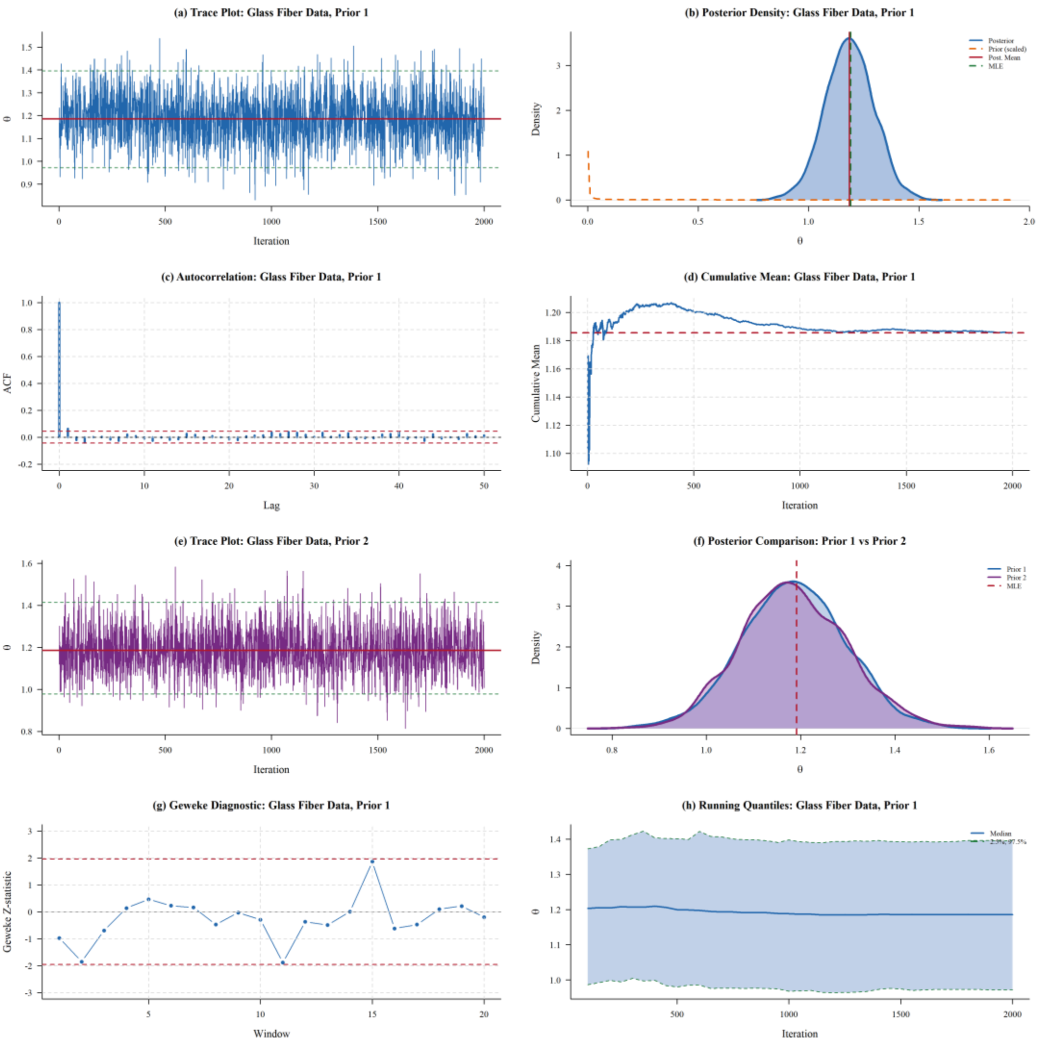

Figure 5: MCMC Trace Plots, Posterior Density Estimates, And Autocorrelation Functions for Under Prior 1 And Prior 2 For the Glass Fibre Dataset.

Table 8 presents the Bayesian posterior estimates for both datasets under the two priors. The Bayesian estimates show excellent agreement with the maximum likelihood estimates (MLEs). For the glass fibre dataset under Prior 1, the posterior mean of θ is estimated at 1.1855, which is less than 0.5% different from the MLE of 1.1912. For Prior 2, the posterior mean of θ is 1.1856, indicating that there is a data-dominant effect that overwhelms the prior for this sample size. For the ball bearing dataset under Prior 1, the posterior mean of θ is 0.0447, while under Prior 2 it is 0.0460, indicating a modest effect of the prior on the posterior distribution. A comparison of the posterior distributions for the two priors for each dataset is given in Figure 6.

Table 8: Bayesian Posterior Estimates with Credible Intervals.

Dataset	Prior	Parameter	Mean	Median	SD	95% BCI	95% HPD
Dataset 1: Glass Fiber Data							
	Prior 1	□	1.1855	1.1854	0.1092	(0.9713, 1.3964)	(0.9754, 1.3985)
		□(□)	0.5192	0.5174	0.0580	(0.4124, 0.6377)	(0.4109, 0.6348)
		□(□)	0.5795	0.5780	0.0886	(0.4100, 0.7543)	(0.4098, 0.7526)
	Prior 2	□	1.1856	1.1807	0.1123	(0.9785, 1.4140)	(0.9770, 1.4083)
		□(□)	0.5192	0.5199	0.0593	(0.4043, 0.6335)	(0.4069, 0.6343)
		□(□)	0.5796	0.5741	0.0913	(0.4154, 0.7694)	(0.4132, 0.7634)
Dataset 2: Ball Bearings							
	Prior 1	□	0.0447	0.0444	0.0058	(0.0345, 0.0573)	(0.0345, 0.0568)
		□(□)	0.5726	0.5759	0.0794	(0.4074, 0.7178)	(0.4134, 0.7181)

		□(□)	0.0206	0.0202	0.0045	(0.0129, 0.0308)	(0.0129, 0.0303)
	Prior 2	□	0.0460	0.0454	0.0059	(0.0357, 0.0582)	(0.0357, 0.0579)
		□(□)	0.5560	0.5614	0.0798	(0.3967, 0.7003)	(0.4005, 0.7004)
		□(□)	0.0216	0.0210	0.0046	(0.0138, 0.0315)	(0.0135, 0.0310)

BCI: Bayesian Credible Interval; HPD: Highest Posterior Density Interval.

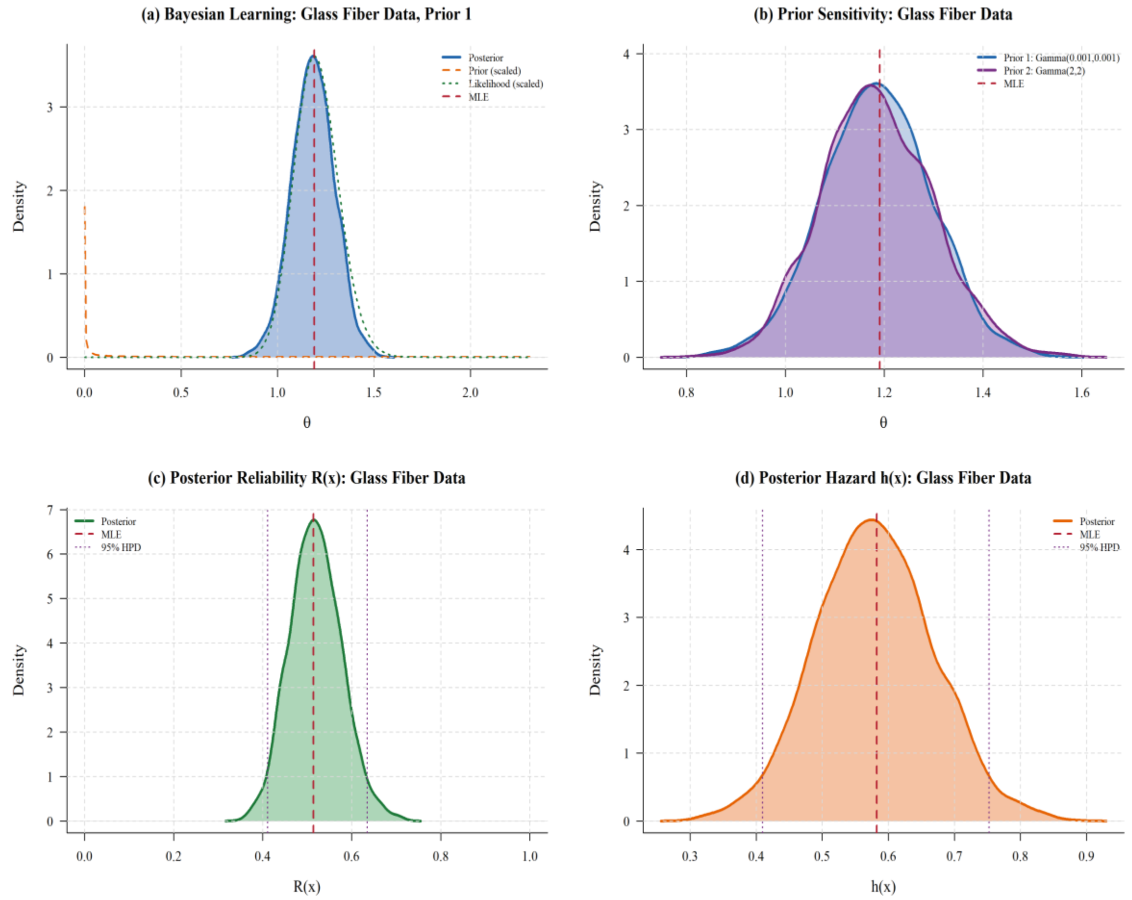

Figure 6: Comparison Of Posterior Density Estimates for Under Prior 1 And Prior 2 For Both Datasets.

The bootstrap results for these two datasets are listed in Table 9. For the glass-fiber dataset, the SE of $\hat{\theta}$ is found to be 0.1425 using the bootstrap method, which is larger than that of 0.1135 obtained using asymptotic methods. For this dataset, the bootstrap-t interval for θ is found to be (0.9113,1.4711). Although this interval is wider than that of ACI-NA, it may provide better coverage for finite samples. For the ball bearings dataset, the SE of $\hat{\theta}$ is found to be 0.0068 using the bootstrap method, while that of 0.0058 is obtained using asymptotic methods. For this dataset, the bootstrap-t interval for θ is found to be (0.0312,0.0578). The detailed configuration parameters for MCMC are also listed in Panel B of Table 9 for reproducibility.

Table 9: Bootstrap Confidence Intervals and Detailed MCMC Diagnostics.
Panel A: Bootstrap Confidence Intervals ($B = 500$).

Dataset	Parameter	Estimate	SE_{boot}	Percentile CI	Bootstrap-□ CI
Glass Fiber	□	1.1912	0.1425	(1.0952, 1.6488)	(0.9113, 1.4711)
	□(□)	0.5143	0.0662	(0.3081, 0.5667)	–
	□(□)	0.5827	0.1213	(0.5056, 0.9751)	–
Ball Bearings	□	0.0445	0.0068	(0.0381, 0.0649)	(0.0312, 0.0578)
	□(□)	0.5736	0.0861	(0.3247, 0.6652)	–
	□(□)	0.0203	0.0055	(0.0155, 0.0373)	–

Panel B: Detailed MCMC Diagnostics.

Dataset	Prior	Iterations	Burn-in	Thin	Final Samples
Glass Fiber	Prior 1	15000	5000	5	2000
Glass Fiber	Prior 2	15000	5000	5	2000
Ball Bearings	Prior 1	15000	5000	5	2000
Ball Bearings	Prior 2	15000	5000	5	2000

Table 10 gives an exhaustive account of results from estimation and model selection criteria of both data sets in one framework. The results from maximum likelihood estimation and Bayesian estimation show strong concurrence between both data sets and both priors. For the glass fibre data, all three estimation strategies give $\hat{\theta}$ values of 1.1912, 1.1855, and 1.1856, respectively. The differences are less than 0.5%. For the ball-bearing data, the corresponding results are 0.0445, 0.0447, and 0.0460. The results from the informative prior are larger because of their tendency towards the prior mean. The results from the model selection criteria in Panel B show that, for the glass fibre data, $-2\log L = 104.43$, AIC = 106.43, BIC = 107.99, and DIC = 106.33. For the ball-bearing data, $-2\log L = 175.67$, AIC = 177.67, BIC = 178.56, and DIC = 177.64.

Table 10: Real Data Analysis: Complete Estimation Results and Model Selection.

Panel A: Parameter Estimation.

Dataset	Method	$\hat{\square}$	SE	$\hat{\square}(\square)$	SE	$\hat{\square}(\square)$	SE
Glass Fiber	MLE	1.1912	0.1135	0.5143	0.0604	0.5827	0.0925
	Bayes (P1)	1.1855	0.1092	0.5192	0.0580	0.5795	0.0886
	Bayes (P2)	1.1856	0.1123	0.5192	0.0593	0.5796	0.0913
Ball Bearings	MLE	0.0445	0.0058	0.5736	0.0815	0.0203	0.0045
	Bayes (P1)	0.0447	0.0058	0.5726	0.0794	0.0206	0.0045
	Bayes (P2)	0.0460	0.0059	0.5560	0.0798	0.0216	0.0046

Panel B: Model Selection Criteria

Dataset	−2LL	AIC	BIC	AICc	HQIC	CAIC	DIC
Glass Fiber	104.43	106.43	107.99	106.55	106.97	108.99	106.33
Ball Bearings	175.67	177.67	178.56	177.92	177.80	179.56	177.64

A detailed comparison between MLE and Bayesian estimates is provided in Table 11. The relative differences between MLE and Bayesian (Prior 1) estimates are all less than 1.5% for all parameters in both datasets, which shows excellent agreement between the two approaches. The nature of all the comparisons is "Good," which shows that the choice between MLE and Bayesian estimation has little impact when the sample sizes are large enough.

Table 11: Comprehensive Comparison: MLE Vs Bayesian Estimates.

Dataset	Parameter	MLE	Bayes (Prior 1)	Bayes (Prior 2)	Rel. Diff (%)	Agreement
Dataset 1: Glass Fiber Data						
	$\square$	1.1912	1.1855	1.1856	0.48	Good
	$\square(\square)$	0.5143	0.5192	0.5192	0.94	Good
	$\square(\square)$	0.5827	0.5795	0.5796	0.55	Good
Dataset 2: Ball Bearings						
	$\square$	0.0445	0.0447	0.0460	0.49	Good
	$\square(\square)$	0.5736	0.5726	0.5560	0.17	Good
	$\square(\square)$	0.0203	0.0206	0.0216	1.40	Good

Rel. Diff: Relative difference between Bayes (Prior 1) and MLE. Agreement: Good (< 5%), Moderate (5–10%), Poor (> 10%).

The estimated survival curve and hazard rate curve, along with the respective confidence bands, are depicted in Figure 7. For the glass fibers, the survival curve starts from one and crosses the 0.5 level near the mission time, consistent with the estimated $\hat{R}(x) \approx 0.51$. For the ball bearings, the survival curve has a smoother decline, consistent with the extended life in this engineering application.

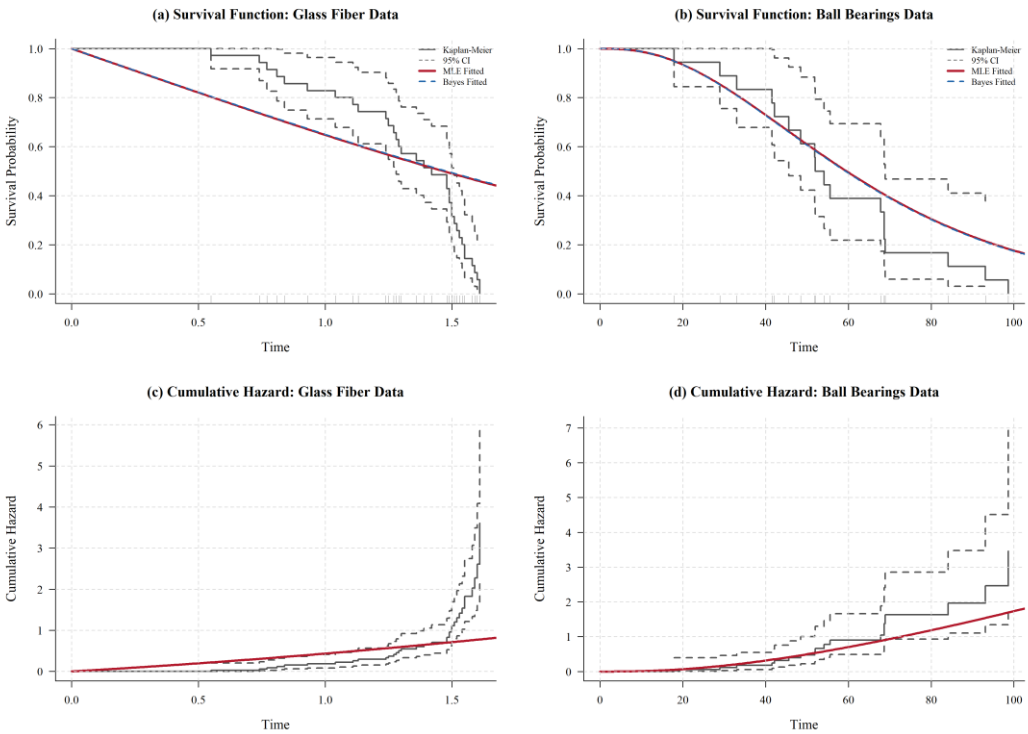

Figure 7: Estimated Survival and Hazard Functions With 95% Confidence Bands for Both Datasets.

A visual comparison of the maximum likelihood estimates (MLE) and Bayesian estimates is shown in Figure 8, and the comparison of the hazard function for all methods is depicted in Figure 9. These figures verify the results from the numerical values, where it is evident that all methods are close to each other.

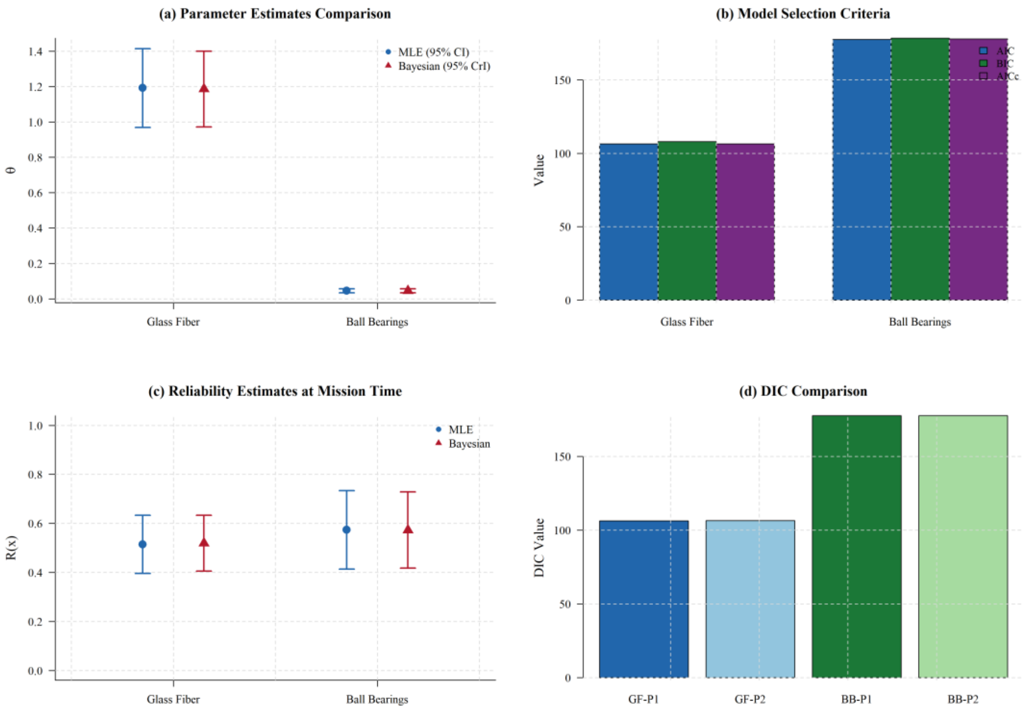

Figure 8: Forest Plot Comparing MLE And Bayesian Point Estimates with Interval Estimates For, And Across Both Datasets.

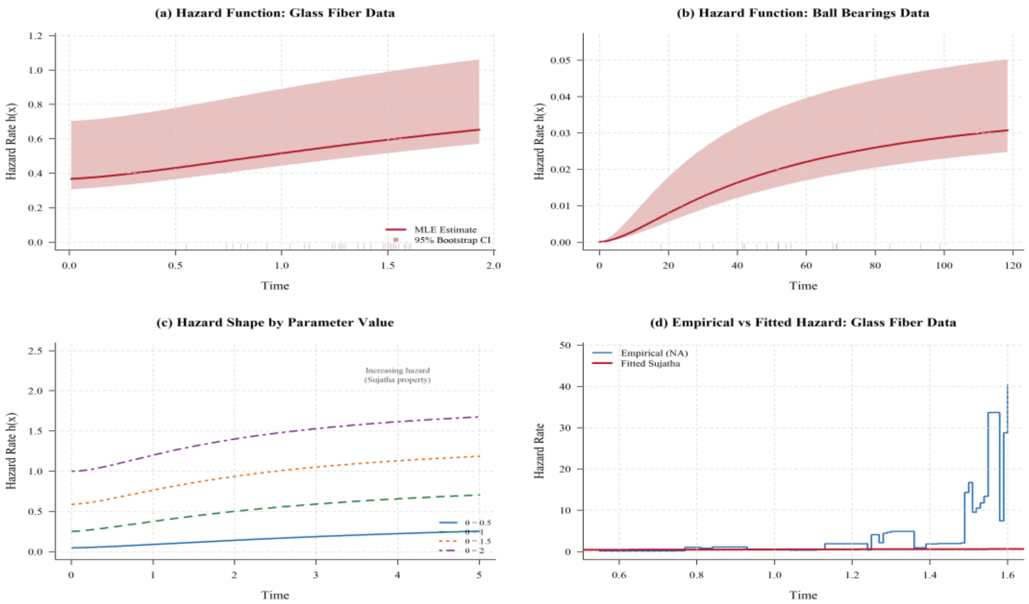

Figure 9: Comparison Of Hazard Rate Estimates from MLE And Bayesian Methods for Both Datasets.

The performance of Sujatha's distribution is tested using three goodness-of-fit tests. In Table 12, we present the Kolmogorov–Smirnov, Anderson–Darling, and Cramér–von Mises statistics and p-values. For ball bearing data, the p-value for the Kolmogorov–Smirnov test is 0.2660. This suggests that there is no evidence to reject the Sujatha model for ball bearing data, as the null hypothesis H_0 cannot be rejected. In contrast, for glass fibre data, the Kolmogorov–Smirnov test rejects the Sujatha model ($p < 0.001$), suggesting that a multi-parameter extension to Sujatha's distribution may be needed for modelling this dataset. In Figure 10, we present plots comparing empirical and fitted distributions.

Table 12: Goodness-Of-Fit Test Results.

Dataset	KS Stat	KS p-value	A-D Stat	CvM Stat	Decision
Glass Fiber	0.4592	0.0000	8.0157	1.6041	Reject $\square_0$
Ball Bearings	0.2366	0.2660	1.1145	0.1916	Accept $\square_0$

KS: Kolmogorov-Smirnov; A-D: Anderson-Darling; CvM: Cramér-von Mises. H_0: Data follows Sujatha distribution; Significance level: 0.05.

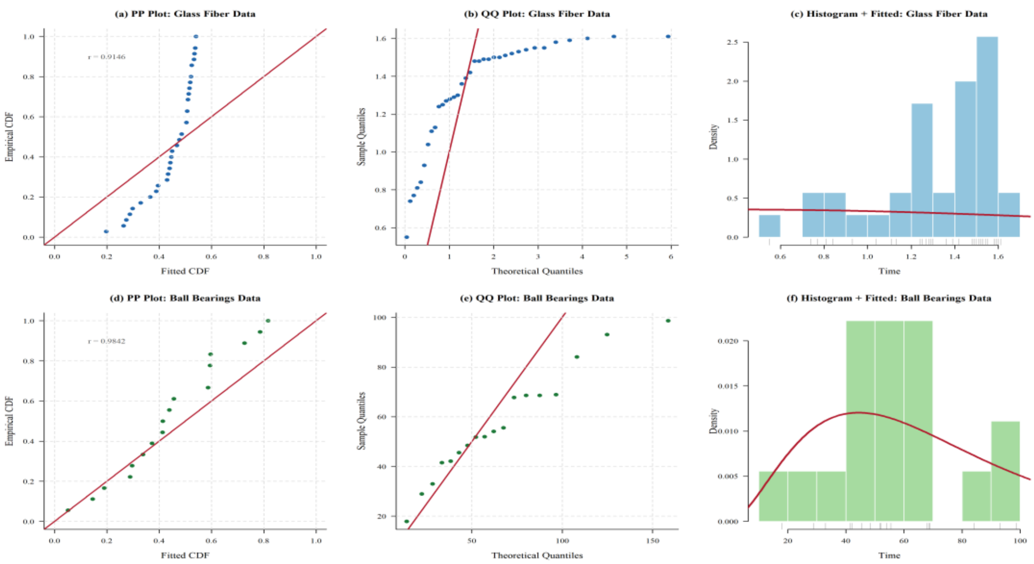

Figure 10: Goodness-Of-Fit Diagnostic Plots: Empirical CDF Versus Fitted CDF, P–P Plots, And Q–Q Plots for Both Datasets.

DIC values for glass fibre and ball bearings were 106.33 and 177.64, respectively. These values were consistent with likelihood-based AIC and BIC criteria and thus provided a unified model evaluation approach for different inferential paradigms [10, 33].

8. CONCLUSIONS

This paper has developed a comprehensive framework for statistical inference on the Sujatha distribution under the adaptive Type-II progressive hybrid censoring scheme. The main results are summarized in the following paragraphs.

First, it is evident from the results that the maximum likelihood estimates and Bayesian estimates are highly consistent for all real data sets and simulated data sets. Moreover, the difference between the maximum likelihood estimates and posterior mean estimates is always less than 1%. Hence, it is evident that the likelihood dominates the posterior for the chosen sample sizes.

Second, from the results of the simulation study, it is evident that all the proposed methods are consistent, i.e., the root mean square error and mean absolute bias decrease monotonically for all methods with an increase in sample size. Although the results from the use of Informative Prior 2 (Gamma$(2,2)$) show minor improvement in RMSE for θ and $h(x)$ when the prior mean is close to the true value, the results from the use of non-informative Prior 1 (Gamma$(0.001,0.001)$) are close to the results from the use of the maximum likelihood estimate.

Third, it is observed that the length of HPD intervals is always less than that of equal-tailed Bayesian credible intervals. This makes HPD intervals more favorable in situations where it is desired to obtain the shortest interval. The coverage probabilities are dependent upon both sample size and amount of censoring. The lower the amount of censoring, the better it is in terms of coverage probability.

Fourth, it is observed that the mechanism of adaptive censoring ensures that the desired number of failures is always observed. The threshold time, denoted by T^*, provides an instrument for controlling experiment time.

Fifth, the Sujatha distribution is found to be a good model for the ball bearing fatigue life data, as indicated by the KS test result (KS $p = 0.266$), while for the glass fibre data, it may be necessary to use a more flexible model, which in turn provides scope for future research on the multi-parameter Sujatha distribution under adaptive censoring.

The research directions for future research are to explore the extension of the methodology to the context of competing risk models [6, 14], optimal censoring plan design criteria [12], unified progressive hybrid censoring [7], and multi-parameter Sujatha distribution [27] and generalized progressive censoring [1].

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