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wbg^#-CLOSED SETS VIA BINARY TOPOLOGY

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ABSTRACT

We present a new class of generalized closed sets in this article, which includes the previously mentioned class, which we name $wbg^{\#}$-closed sets. We also examine the connections between the associated g-closed sets in binary. We also investigate some of the characteristics of $wbg^{\#}$-closed sets and their consequences for topological spaces in general. Using mathematical frameworks, our results provide valuable insights into these sets.

KEYWORDS:

1. INTRODUCTION AND PRELIMINARIES

Among others, Nithyanantha Jothi [4] introduced binary topology. This creative method improves network performance and makes data processing more effective. Binary topology allows systems to optimize information flow, which results in faster and more dependable node-to-node communication. Background information can be found by reading articles [1-14]. This framework facilitates a greater comprehension of the interdependence of various sets by enabling the investigation of links and mappings between them. A number of qualities that are crucial to mathematics, such continuity and compactness, can be discovered by examining these ordered pairings. The class indicated above is part of a new class of generalized closed sets that we introduce in this paper, $wbg^{\#}$-closed sets. We also examine the connections between the associated bg-closed sets. Furthermore, we examine a number of $wbg^{\#}$-closed set characteristics and their consequences for topological spaces in general. Our results show interesting relationships that improve our comprehension of these sets in mathematical contexts.

Therefore, $\mathcal{B}^{\mathcal{M}}$ will be used to refer to a binary topological space $(X, Y, \mathcal{M})$.

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Key words and phrases. $g^{\#}$-closed set, $bg^{\#}$-closed set and $wbg^{\#}$-closed set.

2. WEAKLY BINARY $g^{\#}$-Closed Sets

Definition 2.1 *A weakly binary $g^{\#}$-closed set (i.e., $wbg^{\#}$-closed) set is a subset (C_1, C_2) of $\mathcal{B}^{\mathcal{M}}$ if $cl_b(int_b(C_1, C_2)) \subseteq (G_1, G_2)$ whenever $(C_1, C_2) \subseteq (G_1, G_2)$ and (G_1, G_2) is $b\alpha g$-open. $wbg^{\#}$-open is the complement of $wbg^{\#}$-closed.*

Proposition 2.2 *In $\mathcal{B}^{\mathcal{M}}$, every $bg^{\#}$-closed set is $wbg^{\#}$-closed.*

Remark 2.3 *Proposition 2.2's inverse need not be true, as the example that follows shows.*

Example 2.4 *Assume $X = \{a_1, a_2\}, Y = \{o, p\}$, and $\mathcal{M} = \{(\phi, \phi), (\phi, \{o\}), (\{a_1\}, \{o\}), (X, Y)\}$. In that case, $(\phi, \phi), (\{a_2\}, \{p\}), (X, \{p\})$, and (X, Y) are $bg^{\#}$-closed sets.*

$(\phi, \phi), (\phi, \{p\}), (\{a_1\}, \phi), (\{a_1\}, \{p\}), (\{a_2\}, \phi), (\{a_2\}, \{p\}), (X, \phi), (X, \{p\}), (X, Y)$ are then $wbg^{\#}$-closed sets. In the $\mathcal{B}^{\mathcal{M}}$, the subset $(C_1, C_2) = (\{a_1\}, \{p\})$ is $wbg^{\#}$-closed, but not $bg^{\#}$-closed.

Proposition 2.5 *In $\mathcal{B}^{\mathcal{M}}$, every $wbg^{\#}$-closed set is wbg-closed.*

Proof. (C_1, C_2) is any $wbg^{\#}$-closed set and (G_1, G_2) is any b-open set that contains (C_1, C_2). Then (G_1, G_2) is a $b\alpha g$-open set that contains (C_1, C_2). We have $cl_b(int_b(C_1, C_2)) \subseteq (G_1, G_2)$. Hence, (C_1, C_2) is wbg-closed in $\mathcal{B}^{\mathcal{M}}$.

Remark 2.6 *Proposition 2.5's inverse need not be true, as the example that follows shows.*

Example 2.7 *Assume $X = \{i, j\}, Y = \{f_1, f_2\}$ and $\mathcal{M} = \{(\phi, \phi), (\phi, \{f_1\}), (\{i\}, \{f_1\}), (X, Y)\}$. Then the subsets*

$(\phi, \phi), (\phi, \{f_2\}), (\phi, Y), (\{i\}, \phi), (\{i\}, \{f_2\}), (\{i\}, Y), (\{j\}, \phi), (\{j\}, \{f_1\}), (\{j\}, \{f_2\}), (\{j\}, Y), (X, \phi), (X, \{f_1\}), (X, \{f_2\}), (X, Y)$ are wbg-closed, and $(\phi, \phi), (\phi, \{f_2\}), (\{i\}, \phi), (\{i\}, \{f_2\}), (\{j\}, \phi), (\{j\}, \{f_2\}), (X, \phi), (X, \{f_2\}), (X, Y)$ are $wbg^{\#}$-closed. However, the subset $(\{j\}, \{f_1\})$ is not $wbg^{\#}$-closed, but it is wbg-closed in $\mathcal{B}^{\mathcal{M}}$.

Proposition 2.8 *In $\mathcal{B}^{\mathcal{M}}$, every $wbg^{\#}$-closed set is $wb\pi g$-closed.*

Proof. (C_1, C_2) is any $wbg^{\#}$-closed set and (G_1, G_2) is any $b\pi$-open set that contains (C_1, C_2). Then (G_1, G_2) is a bsg-open set that contains (C_1, C_2). We have $cl_b(int_b(C_1, C_2)) \subseteq (G_1, G_2)$. Hence, (C_1, C_2) is $wb\pi g$-closed in $\mathcal{B}^{\mathcal{M}}$.

Remark 2.9 *Proposition 2.8's inverse need not be true, as the example that follows shows.*

Example 2.10 *Assume $X = \{f, g\}, Y = \{k_1, k_2\}$ and*

$\mathcal{M} = \{(\phi, \phi), (\phi, \{k_1\}), (\phi, \{k_2\}), (\phi, Y), (\{f\}, \{k_1\}), (\{f\}, Y), (\{g\}, \{k_1\}), (\{g\}, Y), (X, \{k_1\}), (X, Y)\}$, Then, $(\phi, \phi), (\phi, \{k_2\}), (\{f\}, \phi), (\{g\}, \phi), (\{g\}, \{k_2\}), (X, \phi), (X, \{k_1\}), (X, \{k_2\}), (X, Y)$ are $wbg^{\#}$-closed, and $wb\pi g$-closed set is $\mathbb{P}(X) \times \mathbb{P}(Y)$. Then the subset $(\{f\}, Y)$ is $wb\pi g$-closed, but not $wbg^{\#}$-closed in $\mathcal{B}^{\mathcal{M}}$.

Proposition 2.11 *In $\mathcal{B}^{\mathcal{M}}$, every $wbg^{\#}$-closed set is $brwg$-closed.*

Proof. Here (C_1, C_2) is any $wbg^{\#}$-closed set and (G_1, G_2) is any br-open set containing (C_1, C_2). Then (G_1, G_2) is an $b\alpha g$-open set containing (C_1, C_2). We have $cl_b(int_b(C_1, C_2)) \subseteq (G_1, G_2)$. Thus, (C_1, C_2) is $brwg$-closed in $\mathcal{B}^{\mathcal{M}}$.

Remark 2.12 *Proposition 2.11's inverse need not be true, as the example that follows shows.*

Example 2.13 *Let $X = \{r, s\}$, $Y = \{i, j\}$ and $\mathcal{M} = \{(\phi, \phi), (\phi, \{j\}), (\{r\}, \{i\}), (\{r\}, Y), (X, Y)\}$. Then the subsets*

$(\phi,\phi),(\phi,\{i\}),(\phi,Y),(\{r\},\phi),(\{s\},\phi),(\{s\},\{i\}),(\{s\},\{j\}),(\{s\},Y),(X,\phi),(X,\{i\}),(X,\{j\}),(X,Y)$ *are* $wbg^{\#}$*-closed and* $(\phi,\phi),(\phi,\{i\}),(\phi,Y),(\{r\},\phi),(\{r\},\{j\}),(\{r\},Y),(\{s\},\phi),(\{s\},\{i\}),(\{s\},\{j\}),(\{s\},Y),(X,\phi),(X,\{i\}),(X,\{j\}),(X,Y)$ *are* $brwg$*-closed. The subset* $(\{r\},\{j\})$ *is then* $brwg$*-closed but not* $wbg^{\#}$*-closed in* $\mathcal{B}^{\mathcal{M}}$.

Theorem 2.14 *A subset* (C_1,C_2) *of* $\mathcal{B}^{\mathcal{M}}$ *is* $wbg^{\#}$*-closed if it is both* b*-closed and* bg*-closed.*

Proof. Let (C_1,C_2) be bg-closed set in $\mathcal{B}^{\mathcal{M}}$ and (G_1,G_2) be any b-open set containing (C_1,C_2) in $\mathcal{B}^{\mathcal{M}}$. Then $(G_1,G_2)\supseteq cl_b(C_1,C_2)\supseteq cl_b(int_b(cl_b(C_1,C_2)))$. Since (C_1,C_2) is b-closed, $(G_1,G_2)\supseteq cl_b(int_b(C_1,C_2))$ and hence $wbg^{\#}$-closed in $\mathcal{B}^{\mathcal{M}}$.

Theorem 2.15 *It is b-closed if a subset* (C_1,C_2) *of* $\mathcal{B}^{\mathcal{M}}$ *is both b-open and* $wbg^{\#}$*-closed in* $\mathcal{B}^{\mathcal{M}}$.

Proof. Since (C_1,C_2) is both b-open and $wbg^{\#}$-closed in $\mathcal{B}^{\mathcal{M}}$, $(C_1,C_2)\supseteq cl_b(int_b(C_1,C_2))=cl_b(C_1,C_2)$ and hence (C_1,C_2) is b-closed in $\mathcal{B}^{\mathcal{M}}$.

Corollary 2.16 *It is both br-open and br-closed in* $\mathcal{B}^{\mathcal{M}}$ *if a subset* (C_1,C_2) *of* $\mathcal{B}^{\mathcal{M}}$ *is both b-open and* $wbg^{\#}$*-closed.*

Theorem 2.17 *Assume* $(C_1,C_2)\subseteq(X,Y)$ *is b-open and* $\mathcal{B}^{\mathcal{M}}$ *is a space. If and only if* (C_1,C_2) *is* $bg^{\#}$*-closed, then* (C_1,C_2) *is* $wbg^{\#}$*-closed.*

Proof. Let (C_1,C_2) be $bg^{\#}$-closed in $\mathcal{B}^{\mathcal{M}}$. By Proposition 2.2, it is $wbg^{\#}$-closed. Conversely, let (C_1,C_2) be $wbg^{\#}$-closed in $\mathcal{B}^{\mathcal{M}}$. Since (C_1,C_2) is b-open, by Theorem 2.15, (C_1,C_2) is b-closed. Hence (C_1,C_2) is $bg^{\#}$-closed in $\mathcal{B}^{\mathcal{M}}$.

Theorem 2.18 *If* (C_1,C_2) *of* $\mathcal{B}^{\mathcal{M}}$ *is* $wbg^{\#}$*-closed, then there isn't a non-empty* $b\alpha g$*-closed set in* $cl_b(int_b(C_1,C_2))-(C_1,C_2)$.

Proof. Let (E_1,E_2) be an $b\alpha g$-closed set in $\mathcal{B}^{\mathcal{M}}$ such that $(E_1,E_2)\subseteq cl_b(int_b(C_1,C_2))-(C_1,C_2)$. Since $(E_1,E_2)^c$ is $b\alpha g$-open and $(C_1,C_2)\subseteq(E_1,E_2)^c$, from the definition of $wbg^{\#}$-closedness it follows that $cl_b(int_b(C_1,C_2))\subseteq(E_1,E_2)^c$. i.e., $(E_1,E_2)\subseteq(cl_b(int_b(C_1,C_2)))^c$. Hence $(E_1,E_2)\subseteq(cl_b(int_b(C_1,C_2)))\cap(cl_b(int_b(C_1,C_2)))^c=(\phi,\phi)$.

Proposition 2.19 *If a subset* (C_1,C_2) *of* $\mathcal{B}^{\mathcal{M}}$ *is b-nowhere dense, then it is* $wbg^{\#}$*-closed.*

Proof. $int_b(C_1,C_2)\subseteq int_b(cl_b(C_1,C_2))$ and (C_1,C_2) are b-nowhere dense, $int_b(C_1,C_2)=(\phi,\phi)$. Therefore $cl_b(int_b(C_1,C_2))=(\phi,\phi)$ and hence (C_1,C_2) is $wbg^{\#}$-closed in $\mathcal{B}^{\mathcal{M}}$.

Remark 2.20 *Proposition 2.19's inverse need not be true, as the example that follows shows.*

Example 2.21 *Let* $X=\{r,s\}$, $Y=\{i,j\}$, *and*

$\mathcal{M}=\{(\phi,\phi),(\phi,\{i\}),(\phi,\{j\}),(\phi,Y),(\{r\},\{1\}),(\{r\},Y),(\{s\},\{j\}),(\{s\},\{j\}),(\{s\},Y),(\{s\},Y),(\{s\},Y),(\{s\},Y),(\{s\},Y),((X,Y)\}$. *Then* $(\phi,\phi),(\{r\},\phi),(\{r\},\{i\}),(\{s\},\phi),(\{s\},\phi),(X,\phi),(X,\{i\}),(X,\{j\}),(X,\{j\}),(X,Y)$ *are* $wbg^{\#}$*-closed. The subset* $(\{r\},\{i\})$ *in the space* $\mathcal{B}^{\mathcal{M}}$ *is* $wbg^{\#}$*-closed set, but not b-nowhere dense.*

Remark 2.22 *In* $\mathcal{B}^{\mathcal{M}}$, $wbg^{\#}$*-closedness and* bs*-closedness are independent of one another, as illustrated by the following example.*

Example 2.23 *In Example 2.13, then bs-closed set are* $(\phi,\phi),(\phi,\{s\}),(\{i\},\{r\}),(\{j\},\phi),(\{j\},\{s\}),(X,\{r\}),(X,Y)$ *, we have the set* $(\{i\},\phi)$ *is* $wbg^{\#}$*-closed set but not* bs*-closed and also the set* $(\{i\},\{r\})$ *is* bs*-closed set but not* $wbg^{\#}$*-closed in* $\mathcal{B}^{\mathcal{M}}$.

Remark 2.24 *These relationships are depicted in the diagram, where* $E_1\to E_2$ *indicates that* E_1 *implies* E_2*, but not the other way around.*

Diagram

b-closed	→	**$wbg^{\#}$-closed**	→	**wbg-closed**
				↓
		$wb\pi g$-closed	←	$brwg$-closed

Definition 2.25 *If* $(C_1,C_2)^c$ *is* $wbg^{\#}$*-closed, then* (C_1,C_2) *is a subset of* $\mathcal{B}^{\mathcal{M}}$ *that is called a* $wbg^{\#}$*-open set.*

Proposition 2.26 $\mathcal{B}^{\mathcal{M}}$ *contains the following statements.*

1. Every $bg^{\#}$-open set is also $wbg^{\#}$-open, but not the other way around.
2. Every bg-open set is also $wbg^{\#}$-open, but not the other way around.

Theorem 2.27 $wbg^{\#}$*-open is a subset* (C_1,C_2) *of a* $\mathcal{B}^{\mathcal{M}}$*. When* $(E_1,E_2)\subseteq(C_1,C_2)$ *and* (E_1,E_2) *are* $b\alpha g$*-closed, if* $(E_1,E_2)\subseteq int_b(cl_b(C_1,C_2))$.

Proof. Let any $wbg^{\#}$-open be (C_1,C_2). Consequently, $(C_1,C_2)^c$ is $bwg^{\#}$-closed. Let (C_1,C_2) contain a $b\alpha g$-closed set (E_1,E_2). A $b\alpha g$-open set that contains $(C_1,C_2)^c$ is then $(E_1,E_2)^c$. The $cl_b(int_b((C_1,C_2)^c))\subseteq(E_1,E_2)^c$ is $wbg^{\#}$-closed since $(C_1,C_2)^c$. It follows that $(E_1,E_2)\subseteq int_b(cl_b(C_1,C_2))$.

When $(E_1,E_2)\subseteq(C_1,C_2)$ and (E_1,E_2) are $b\alpha g$-closed, on the other hand, we assume that $(E_1,E_2)\subseteq int_b(cl_b(C_1,C_2))$. It follows that $(E_1,E_2)^c$ is a $b\alpha g$-open set that contains $(C_1,C_2)^c$ and $(E_1,E_2)^c\supseteq(int_b(cl_b(C_1,C_2)))^c$. Consequently, $(E_1,E_2)^c\supseteq cl_b(int_b((C_1,C_2)^c)$ does not exist. For this reason, $(C_1,C_2)^c$ is $wbg^{\#}$-closed, and (C_1,C_2) is $wbg^{\#}$-open.

3. CONCLUSION

This work presents a fresh type of generalized closed sets, which includes the previously mentioned class, which is named $wbg^{\#}$-closed sets. The relationships between the associated bg-closed sets were also investigated. To further illustrate the properties of $wbg^{\#}$ -closed sets and their significance in topology overall, we have included several examples. Our results suggest potential applications in various mathematical fields and will help guide future research on generalized closed set theory.

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