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ENHANCING CRITICAL THINKING AND MATHEMATICAL COMMUNICATION SKILLS THROUGH REFLECTIVE PROBLEM-BASED LEARNING

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ABSTRACT

Critical thinking and mathematical communication are fundamental competencies in twenty first century mathematics education, particularly for prospective primary school teachers. This study aims to examine the effectiveness of reflective problem based learning, referred to as R-PBL, in enhancing students' critical thinking and mathematical communication skills. A quantitative approach employing a quasi-experimental design was adopted. The participants comprised 234 undergraduate students enrolled in the Primary School Teacher Education Programme at PGRI University Palembang, who were distributed across six classes, including three experimental classes and three control classes. The experimental group engaged in learning activities through R-PBL, whereas the control group received problem based learning implemented through lecture based instruction and classroom discussion. Data were collected using critical thinking and mathematical communication tests administered at the pretest and posttest stages. Data analysis involved descriptive statistics, N-gain calculations, and the Mann Whitney nonparametric test. The findings revealed that students in the experimental group demonstrated a significantly greater improvement in both critical thinking and mathematical communication skills compared to those in the control group. The experimental group achieved

moderate to high N-gain scores, whereas the control group showed lower improvement. These results indicate that the integration of authentic problem solving with structured mathematical reflection strengthens higher order thinking processes and enhances the quality of students' mathematical communication. It is therefore concluded that reflective problem based learning represents an effective and pedagogically relevant approach for mathematics instruction in higher education, particularly within programmes designed to prepare prospective primary school teachers.

KEYWORDS: Reflective Problem Based Learning; Critical Thinking; Mathematical Communication; Mathematics Education.

1. INTRODUCTION

Education constitutes a strategic and systematically designed process aimed at developing human potential in a holistic manner, encompassing intellectual, emotional, social, moral, and spiritual dimensions. In the global context, education is no longer understood merely as the transmission of knowledge, but rather as a vehicle for cultivating higher order thinking skills that enable individuals to adapt, innovate, and make rational decisions in response to the complexities of contemporary life (Schwartz, 2024; Reimers, 2020; Hassan et al., 2024). Rapid developments in information technology, automation, and artificial intelligence have fundamentally transformed the ways in which people work, interact, and learn (Khogali & Mekid, 2023; Bauer & Vocke, 2019; Marlow, 2025; Jarrahi, 2019; Kumar, 2023). Knell (2021), Castells (2022) and Hilbert (2020) argue that global society has evolved from the agrarian era to the industrial era and subsequently to the era of knowledge and global information networks. Within this context, education is required not only to produce graduates who possess strong conceptual knowledge, but also to cultivate critical thinking skills, communication skills, collaboration skills, and the capacity to reflect upon and evaluate their own cognitive processes.

Higher education plays a strategic role in preparing adaptive and competitive human resources (Alfawaire & Atan, 2021; Chavez et al., 2023). In Indonesia, this expectation is formally articulated through the Indonesian Qualifications Framework (IQF), which positions bachelor degree graduates at level six. Graduates are expected to be capable of applying knowledge independently to solve problems, making decisions based on the analysis of data and information, and assuming responsibility for the outcomes of both individual and group work (Presidential Decree No. 8 of 2012). Critical thinking and communication skills are no longer additional competencies but the core of learning outcomes for higher education graduates.

Consistent with this perspective, numerous studies emphasise that twenty first century skills include critical thinking, communication, creativity, collaboration, and information and technology literacy (Thornhill-Miller et al., 2023; Kocak et al., 2021; Dilekçi & Karatay, 2023; Weber & Greiff, 2023). In the digital era, critical thinking and mathematical communication serve as essential foundations for new forms of literacy, given that information is increasingly dynamic, complex, and often ambiguous (Kivunja, 2015) (Bredberg, 2020; Szabo et al., 2020; Arisoy & Aybek, 2021; Kozyreva et al., 2022;

François et al., 2020). Critical thinking and mathematical communication skills are essential requirements in mathematics education at the higher education level, particularly for prospective primary school teachers. Various national and international indicators show that the quality of Indonesian students' higher-order thinking skills is still low. The UNDP's Human Development Index (HDI) report ranks Indonesia in an unsatisfactory position, partly reflecting the quality of education and human resource competencies (United Nations Development Programme, 2022). The results of national and international assessments, such as the National Assessment (NA) and the Programme for International Student Assessment (PISA), also indicate the weakness of students' abilities in solving non-routine problems that require deep reasoning and analysis (OECD, 2017; Shawan et al., 2021; Sánchez-Barbero et al., 2020; Muhaimin et al., 2024; Kusmaryono & Kusumaningsih, 2023). This situation points to a fundamental issue in learning practices, particularly in mathematics education, which has tended to emphasise procedural mastery and memorisation. Consequently, students are insufficiently trained to develop critical thinking skills and to communicate mathematical ideas in an argumentative, systematic, and reflective manner.

Although critical thinking and mathematical communication are widely recognised as key competencies for the twenty first century, there remains conceptual and pedagogical debate regarding effective strategies for developing these skills within mathematics learning. Active learning approaches, such as Problem Based Learning, are often regarded as capable of enhancing higher order thinking through student engagement in contextual problem solving (Aba-Oli et al., 2024; Razak et al., 2022; Karan & Brown, 2022; Karan & Brown, 2022; Kardoyo et al., 2020). However, several studies indicate that the implementation of Problem Based Learning does not consistently lead to significant improvements when it is not supported by pedagogical designs that explicitly encourage deep reflection and evaluation of students' thinking processes (Jamiat, 2018; Razak et al., 2022; Sánchez-García & Reyes-De-Cózar, 2025; Pudholi et al., 2025). Another ongoing debate concerns the orientation of mathematics instruction, which frequently prioritises final answers over cognitive processes. Learning practices that focus primarily on correctness tend to marginalise reflective thinking, which is in fact central to the development of critical reasoning skills (Edzgeradze, 2023). As a result, students may be able to solve routine problems but encounter

difficulties when faced with tasks that demand non routine reasoning, contextual interpretation, and logical communication of mathematical ideas. In this regard, reflective thinking, which integrates reflection and critical evaluation within decision making processes, remains relatively underrepresented in mathematics learning models, particularly in higher education programmes for prospective teachers.

A review of the literature reveals that although Problem Based Learning has been shown to be effective in improving problem solving and critical thinking skills, studies that position mathematical communication as a primary outcome remain limited. Moreover, the explicit integration of reflective or refractive dimensions within Problem Based Learning has rarely been examined empirically, especially in the context of higher education and basic geometry instruction. Refractive thinking contributes to the enrichment of problem solving through critical evaluation, the exploration of alternative solutions, and rational decision making (Fatoni et al., 2021; Kuncoro et al., 2025). Empirical studies that simultaneously investigate the effectiveness of reflective oriented Problem Based Learning on both critical thinking and mathematical communication, particularly among students of Primary School Teacher Education programmes, remain scarce. This gap is particularly significant given the strategic role of primary school teachers in establishing students' foundational mathematical thinking from an early age.

Recent research indicates that critical thinking and mathematical communication can be enhanced through learning environments that emphasise contextual problem solving and social interaction. Widiawati et al. (2018) reported that Problem Based Learning improved students' higher order thinking skills, although mathematical communication was not examined in depth. Similarly, Supena et al. (2021) found that learning approaches based on the four C framework had a positive impact on learning achievement, yet did not explicitly integrate reflective processes into instructional design. Studies on reflective thinking in mathematics have largely focused on analysing individual cognitive processes during problem solving (Prayitno et al., 2014) and have not been widely operationalised as a comprehensive pedagogical framework in classroom practice. This condition highlights a significant research opportunity to develop and empirically test learning models that integrate Problem Based Learning with a structured mathematical reflective orientation.

A number of studies support the importance of developing critical thinking and mathematical communication skills in mathematics learning. Wang and Abdullah (2024), Sosa-Gutierrez et al. (2023), Arisoy and Aybek (2021), Monteleone et al. (2023) and Hattori et al. (2025) emphasise that critical thinking in mathematics involves the ability to analyse arguments, evaluate, and make decisions based on logical reasoning. In parallel, National Council of Teachers of Mathematics (NCTM) (2020), Khadka (2024), Uyen et al. (2021) and Serbin and Wawro (2022) highlight the essential role of mathematical communication in fostering conceptual understanding and mathematical reasoning. Nevertheless, most of these studies have not comprehensively explored how these competencies can be developed simultaneously through a structured, reflection based learning model. Accordingly, there is a clear need for pedagogical approaches that explicitly integrate problem solving, reflection, and communication within a unified instructional design.

Based on theoretical and empirical studies, this study formulates the hypothesis that Problem Based Learning oriented towards mathematical reflective thinking is significantly more effective in enhancing students' critical thinking and mathematical communication skills than conventional learning approaches. Accordingly, this research aims to examine the effectiveness of the Reflective Problem Based Learning model in improving the critical thinking and mathematical communication skills of students enrolled in the Basic Geometry course within the Primary School Teacher Education programme. This study is expected to contribute theoretically to the development of reflection based mathematics learning models and practically to lecturers and higher education institutions in designing instructional practices that align with contemporary educational demands.

2. METHODOLOGY

2.1. Research Design

This study employed a quasi experimental approach using a non equivalent control group design to examine the effectiveness of Reflective Problem Based Learning (RPBL) in improving students' critical thinking and mathematical communication skills. This design was selected because the researcher was unable to fully randomise participants, while still seeking to conduct a systematic comparison of learning outcomes between the group receiving the RPBL intervention and the group exposed to conventional learning. The

research design is presented in figure 1.

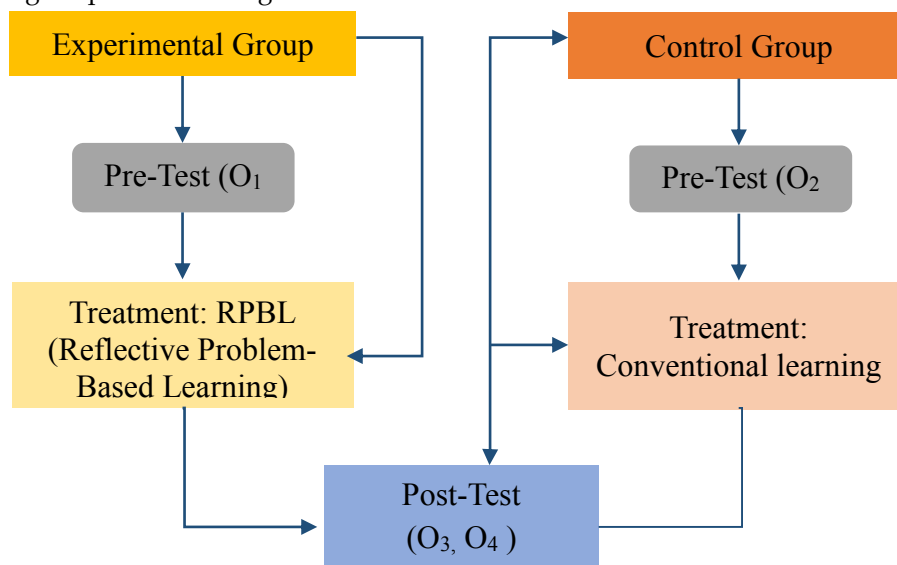


Figure 1: Quasi-Experimental Research Design.

2.2. Research Participants

The research participants comprised 234 undergraduate students enrolled in the Primary School Teacher Education Study Programme at PGRI University Palembang. The participants were distributed across six intact classes, consisting of three experimental classes and three control classes. The experimental group engaged in learning activities using Reflective Problem Based Learning, whereas the control group participated in learning through a Problem Based Learning approach primarily implemented via lectures and classroom discussions.

2.3. Learning Treatment

2.3.1. Reflective Problem-Based Learning (RPBL)

Reflective Problem Based Learning (RPBL) represents an extension of Problem Based Learning that is enriched with a mathematical reflective orientation. The learning process was implemented through the following sequential stages:

1. Presentation of contextual mathematical problems
2. Collaborative problem analysis
3. Exploration and problem solving
4. Structured mathematical reflection, including:
 - evaluation of solution strategies,
 - justification of solutions,
 - communication of mathematical ideas orally and in writing
5. Presentation and discussion of reflection results

2.3.2. Control Group Learning

The control group followed conventional instructional approaches that emphasised lecturer explanations, limited classroom discussions, and problem solving activities conducted without explicit or structured stages of mathematical reflection.

2.4. Research Instruments

This study used two main instruments:

1. Critical Thinking Skills Test, which measures the ability to:

- interpretation,
- analysis,
- evaluation,
- inference,
- mathematical explanation.

2. Mathematical Communication Test, which measures the ability to:

- representing mathematical ideas (diagrams, tables, graphs),
- expressing mathematical symbols and models,
- communicating logical arguments in writing.

Both instruments were administered as pre-tests and post-tests to students in the experimental and control groups. Prior to implementation, the instruments underwent expert content validation and empirical testing. Content validity was established using Aiken's V index ($V = 1.00$), indicating excellent agreement among experts. Construct validity was confirmed through confirmatory factor analysis, demonstrating acceptable model fit (χ^2 p-value = 0.13; CFI ≥ 0.90 ; TLI ≥ 0.90 ; RMSEA = 0.03; SRMR ≤ 0.05). Reliability analysis based on the test information function

yielded coefficients of 0.97 for the Critical Thinking Skills Test and 0.96 for the Mathematical Communication Test, indicating high measurement precision.

2.5. Data Collection Techniques

Research data was collected through:

1. critical thinking and mathematical communication skills tests,
2. observations of learning implementation in the experimental group,
3. documentation of learning implementation.

2.6. Data Analysis Techniques

Data analysis is carried out through the following stages:

2.6.1. Descriptive Analysis

Descriptive analysis was employed to obtain the mean, standard deviation, minimum score, maximum score, and N gain values in order to determine the extent of improvement in students' critical thinking and mathematical communication skills.

2.6.2. Statistical Assumption Test

Before inferential analysis, the following are conducted:

1. a normality test using Kolmogorov–Smirnov,
2. homogeneity test using Levene's Test.

The data were considered to meet the statistical assumptions when the significance value exceeded 0.05.

2.6.3. Inferential Analysis (MANOVA)

To examine the effect of Reflective Problem-Based Learning on students' critical thinking and mathematical communication skills simultaneously, Multivariate Analysis of Variance (MANOVA) was employed with a significance level of $\alpha = 0.05$. MANOVA was chosen because the study involved two related dependent variables across two groups. Prior to the analysis, assumption tests of normality and homogeneity were conducted. When the assumptions were satisfied, MANOVA was applied as the primary analysis. However, when normality assumptions were not fully met, the Mann–Whitney U test was used as a non-parametric alternative to compare differences between groups, ensuring analytical robustness. When MANOVA indicated significant differences, follow-up univariate ANOVA was conducted to examine the effect on each dependent variable separately.

3. RESULTS

3.1. Descriptive Statistics of Critical Thinking and Mathematical Communication Skills

Descriptive statistical analysis was conducted to provide an initial overview of students' critical thinking and mathematical communication skills before and after the implementation of Reflective Problem Based Learning. This analysis included the minimum score, maximum score, mean, and standard deviation for both the experimental and control groups.

The results of the descriptive analysis for the experimental group are presented in Table 1.

Table 1: Descriptive Statistics of Critical Thinking and Mathematical Communication Skills in the Experimental Group.

Data	N	Minimum	Maximum	Mean	Standard Deviation
Pre-test Critical Thinking	119	48.75	77.5	61.1660	8.35794
Post-test Critical Thinking	119	75.00	100	87.9937	6.36383
Pre-test in Mathematical Communication	119	42.47	76.71	60.5359	7.96517
Post-test Mathematical Communication	119	68.49	100.00	86.7490	6.7685

Based on Table 1, a substantial descriptive improvement was observed in both research variables following students' participation in Reflective Problem Based Learning. The mean score for critical thinking increased from 61.17 at the initial stage to 87.99 at the final stage. This increase was accompanied by a reduction in the standard deviation, indicating that the distribution of students' abilities became more homogeneous.

A similar pattern was identified for mathematical

communication skills. The mean score increased from 60.54 to 86.75, accompanied by a decrease in the standard deviation. These findings indicate that Reflective Problem Based Learning not only improves average student achievement but also contributes to a more even distribution of critical thinking and mathematical communication skills among students.

The results of the descriptive analysis for the control group are presented in Table 2.

Table 2: Descriptive Statistics of Critical Thinking**and Mathematical Communication Skills in the Control Group.**

Data	N	Minimum	Maximum	Mean	Standard Deviation
Pre-test Critical Thinking	115	51.25	73.75	59.2717	7.13087
Post-test Critical Thinking	115	60.00	96.25	76.5978	5.94554
Pre-test in Mathematical Communication	115	43.84	76.71	59.4368	6.37490
Post-test Mathematical Communication	115	57.53	91.78	72.6357	8.36584

In contrast, the control group also demonstrated improvement, although with relatively lower achievement levels. The mean critical thinking score increased from 59.27 to 76.60, while the mean score for mathematical communication increased from 59.44 to 72.64. Despite these gains, the post learning scores consistently favoured the experimental group, indicating an advantage for students who received Reflective Problem Based Learning.

3.2. Analysis of Student Ability Improvement (N-Gain)

Table 3: N-Gain Values for Critical Thinking and Mathematical Communication Skills.

Data	N	Minimum	Maximum	Mean	Standard Deviation
N-Critical Thinking Gain	40	0.21	1	0.6659	0.16601
N-Gain Mathematical Communication	40	0.00	1.00	0.6243	0.21596

The average N gain values indicate that the improvement in critical thinking skills reached a moderate to high level, while improvement in mathematical communication skills reached a moderate level.

These findings provide preliminary evidence that Reflective Problem Based Learning is effective in enhancing the two key competencies examined in this study.

A broader N gain analysis was conducted to

Table 4: Descriptive Statistics of N Gain for the Experimental and Control Groups.

Data	N	Minimum	Maximum	Mean	Standard Deviation
N-Gain Critical Thinking Experiment	119	0.21	1	0.6830	0.18678
N-Gain Mathematical Communication Experiment	119	0.00	1	0.6561	0.18553
N-Gain Critical Thinking Control	115	-0.08	0.86	0.4166	0.15701
N-Gain Mathematical Communication Control	115	-0.30	0.82	0.3119	0.23041

These results demonstrate that the experimental group achieved substantially greater improvement than the control group across both variables.

The higher N gain values in the experimental group reinforce earlier findings that Reflective Problem Based Learning exerts stronger effects on the development of students' critical thinking and mathematical communication skills.

To assess the level of learning effectiveness more comprehensively, an analysis of student ability improvement was conducted using the Normalised Gain (N-gain) index. This analysis aimed to see the proportion of ability improvement achieved by students after the learning treatment.

The initial N-gain calculation results (Table 3) show that the average increase in critical thinking skills is in the moderate to high category ($M = 0.67$), while the increase in mathematical communication is in the moderate category ($M = 0.62$).

compare learning effectiveness between the experimental and control groups. The experimental group achieved an average N gain of 0.68 for critical thinking and 0.66 for mathematical communication, both of which fall within the moderate to high improvement category.

In contrast, the control group achieved N gain values of 0.42 for critical thinking and 0.31 for mathematical communication, which correspond to low to moderate improvement levels. The results of this analysis are presented in Table 4.

3.3. Test of Equality of Initial Abilities Between Groups

To ensure that differences in post test outcomes were not attributable to disparities in initial abilities, an equivalence test was conducted using pre test scores from both groups.

The results of the pre-test normality test on a

broad scale are presented in Table 5.

Table 5: Pre-Test Normality Test Results.

Data	Model/Group	Kolmogorov-Smirnov ^a	
		df	Sig.
Critical Thinking Skills	PBL-RM/Experiment	119	0
	Conventional	115	0
Mathematical Communication	PBL-RM/Experiment	119	0.005
	Conventional	115	0.000

As shown in Table 5, the significance values are less than 0.05, indicating that the pre test data for both critical thinking and mathematical communication skills are not normally distributed.

Accordingly, a non parametric statistical procedure was required. The Mann Whitney Test was therefore employed to examine equivalence between groups. The results of this analysis are presented in Table 6.

Table 6: Intergroup Equality Test with Mann-Whitney Test.

	Critical Thinking	Communication
Mann-Whitney U	5854.000	6317.000
Wilcoxon W	12,524.000	12,987.000
Z	-1.919	-1.023
Asymp. Sig. (two-tailed)	0.055	0.306

The results of the Mann Whitney analysis indicate that there were no statistically significant differences between the experimental and control groups in critical thinking skills, with p equal to 0.055, or in mathematical communication skills, with p equal to 0.306. These findings confirm that both groups were equivalent at the initial stage, thereby strengthening the inference that differences observed at the final stage can be attributed to the instructional intervention.

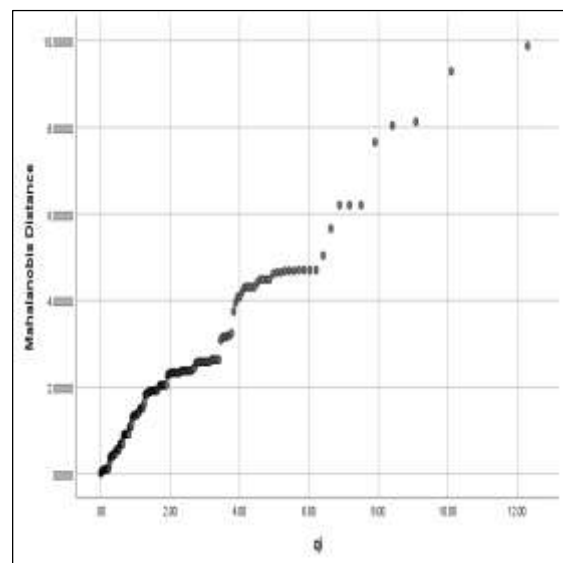
3.4. Multivariate Analysis Assumption Test

Prior to conducting the main multivariate analysis, statistical assumption testing was performed, including assessments of multivariate normality and homogeneity of the variance covariance matrix. These assumptions are essential to ensure the validity of MANOVA results.

Multivariate normality was examined using a scatter plot of Mahalanobis distance against Chi Square values. The null hypothesis was accepted when the data points formed a linear pattern and approximately half of the Mahalanobis distance values were less than or equal to the corresponding Chi Square values. The results of the Mahalanobis distance normality correlation test are presented in Table 7.

Table 7: Mahalanobis Distance Normality Correlation Test.

Correlations			
		Mahalanobis Distance	qi
Mahalanobis Distance	Pearson Correlation	1	0.987**



	Sig. (two-tailed)		0.000
	N	234	234
qi	Pearson Correlation	0.987**	1
	Sig. (two-tailed)	0.000	
	N	234	234

The results indicate a Pearson correlation coefficient of r equal to 0.987 with a significance level of p less than 0.001, reflecting a very strong linear relationship between Mahalanobis distance and Chi Square values. This result confirms that the data meet the assumption of multivariate normality. This

conclusion is further supported by the scatter plot shown in Figure 2, which demonstrates that the data points form a linear pattern along the diagonal line.

Figure 2: Scatter Plot Mahalanobis Distance Normality Test with Chi Square.

Based on Figure 2, it can be seen that the results of the scatter plot tend to form a straight line. It can be concluded that the data is normally distributed in a multivariate manner. This conclusion is also supported by the correlation value analysis. The correlation analysis results can be seen in Table 7, which was discussed earlier. After conducting the normality test, the next step is to conduct a homogeneity test using Box's Test of Equality of Covariance Matrices. Based on the statistical analysis and visualisation results, it can be concluded that the research data meets the assumption of multivariate normality.

Following the normality assessment, homogeneity of the variance covariance matrix was tested using Box's Test of Equality of Covariance Matrices. The results are presented in Table 8.

Table 8: Results of Box's Test of Equality of Covariance Matrices.

Statistics	Value
Box's M	7.530
F	2.487
df1	3
df2	100,634,418.38
Sig.	0.059

The significance value obtained from Box's Test is 0.059, which exceeds the threshold of 0.05. This result indicates that the variance covariance matrices are homogeneous across groups, confirming that the data are suitable for subsequent MANOVA.

3.5. Multivariate Effect of Reflective Problem-Based Learning (MANOVA)

After all statistical assumptions were satisfied, Multivariate Analysis of Variance was conducted to examine simultaneous differences in critical thinking and mathematical communication skills between the experimental and control groups. The MANOVA results are presented in Table 9.

Table 9: MANOVA Test Results.

Statistics	Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared
Pillai's Trace	0.526	128.198	2	231	0.000	0.526
Wilks' Lambda	0.474	128.198	2	231	0.000	0.526

Hotelling's Trace	1,110	128,198	2	231	0	0.526
Roy's Largest Root	1,110	128,198	2	231	0	0.526

The analysis reveals that Wilks' Lambda is statistically significant at p less than 0.001, leading to the rejection of the null hypothesis. This finding indicates a significant multivariate effect of Reflective Problem Based Learning on students' critical thinking and mathematical communication skills. The partial eta squared value of 0.526 suggests that 52.6 percent of the combined variance in both dependent variables is attributable to the instructional intervention, representing a strong and meaningful effect.

3.6. Univariate Effects on Each Variable (Advanced ANOVA)

To examine the effect of Reflective Problem Based Learning on each dependent variable separately, univariate analyses were conducted using Tests of Between Subjects Effects. The results are presented in Table 10.

Table 10: Results of Tests of Between-Subjects Effects.

Dependent Variable	F	Sig	Partial Eta Squared
Critical Thinking	200.034	0.000	0.463
Mathematical Communication	201,916	0.000	0.465

The findings indicate that Reflective Problem Based Learning has a statistically significant effect on critical thinking skills, with F equal to 200.034 and p less than 0.001. The partial eta squared value of 0.463 indicates that 46.3 percent of the variance in critical thinking skills is explained by the learning model. Similarly, mathematical communication skills yielded an F value of 201.916 with p less than 0.001 and a partial eta squared of 0.465, indicating that 46.5 percent of the variance in mathematical communication skills is attributable to Reflective Problem Based Learning. Overall, these results confirm that the learning model exerts a strong and consistent effect on both outcome variables.

3.7. Supporting Visuals

To support the statistical findings, boxplots were employed to visualise the distribution of critical thinking and mathematical communication scores for both groups. These visualisations illustrate the median, interquartile range, and dispersion of scores,

thereby providing additional insight into group differences. Figures 3 and 4 demonstrate that the experimental group exhibits higher median scores and narrower distribution ranges than the control group. This pattern indicates that Reflective Problem Based Learning not only enhances student achievement but also promotes a more equitable distribution of critical thinking and mathematical communication skills.

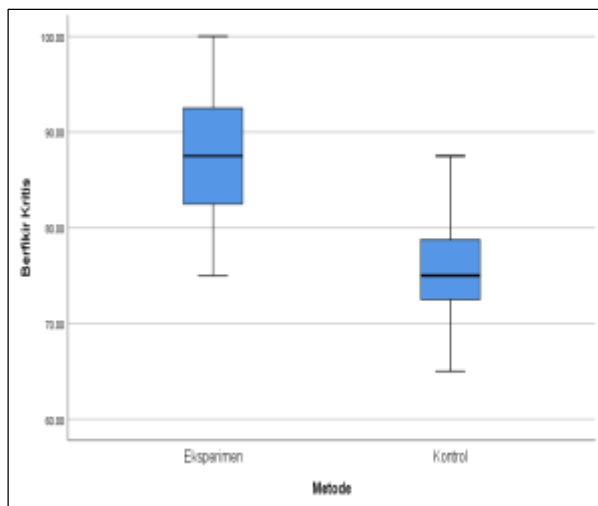


Figure 3: Boxplot of Critical Thinking Skills.

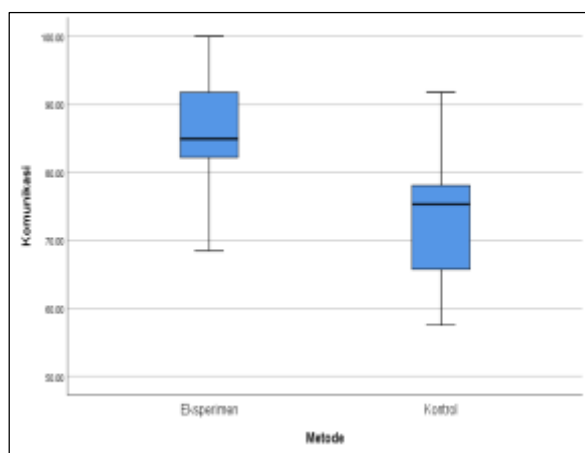


Figure 4: Boxplot of Mathematical Communication.

The visualisation shows that the experimental group has a higher median score and a narrower distribution range than the control group. This pattern indicates that reflective problem-based learning not only improves students' achievement but also promotes the equal distribution of critical thinking and mathematical communication skills.

4. DISCUSSION

Reflective problem based learning demonstrates a significant advantage in enhancing students' critical thinking and mathematical communication skills when compared to conventional instructional

approaches. The quantitative findings indicate that students who participated in learning through the R-PBL approach experienced statistically significant improvements in both competencies. These results support the proposed research hypothesis and indicate that the integration of authentic problem solving and structured mathematical reflection constitutes an effective instructional mechanism for developing higher order thinking skills. Authentic learning practices have been shown to foster critical thinking abilities, including comprehension, comparison, justification, and reflective evaluation of problem solving processes (Dolapcioglu & Doğanay, 2020). Empirical evidence from this study further reveals that students in the experimental group achieved higher mean scores on indicators of analysis, evaluation, and conclusion drawing, which represent critical thinking skills, as well as on indicators of visual representation, mathematical expression, and written argumentation, which represent mathematical communication skills. These findings confirm that reflective problem based learning influences not only cognitive learning outcomes, but also the quality of students' reasoning processes and mathematical communication.

The effectiveness of reflective problem based learning can be interpreted through the theoretical lenses of social constructivism and cognitive reflection theory. Within this learning framework, students actively construct knowledge through social interaction, collaborative discussion, and systematic reflection on the solutions they generate. The reflective component enables students to revisit problem solving strategies, evaluate the validity of mathematical arguments, and connect newly acquired concepts with prior knowledge (Kuncoro et al., 2025; Susanti, 2021; Kholid et al., 2022; Kholid et al., 2024). By embedding reflection within the problem based learning cycle, students' metacognitive processes are strengthened, encouraging attention not only to final answers, but also to reasoning pathways, representational accuracy, and logical coherence. This aligns with research suggesting that critical thinking develops most effectively when learners are consistently provided with opportunities to reflect on their own thinking in a structured and deliberate manner (Lee et al., 2021; Rivas et al., 2022; Darcie et al., 2024; Wheeley et al., 2022; Campo et al., 2023; Bassachs et al., 2020).

The findings further indicate that reflective problem based learning has a substantial impact on the development of students' mathematical communication skills. In line with previous studies,

mathematical communication develops optimally when students are engaged in activities that require elaboration of ideas, acknowledgement of alternative viewpoints, and rational justification of arguments (Howe et al., 2019; Uyen et al., 2021; Tong et al., 2021; Tong & Uyen, 2022; Rico et al., 2023). Within the R-PBL framework, these communicative practices are embedded throughout all stages of learning, ranging from problem presentation to final reflective evaluation of solutions. These results are consistent with earlier findings emphasising that group discussion, data analysis, and presentation of outcomes represent learning activities rich in mathematical communication practices (Wan Husin et al., 2016; Wijayanto et al., 2023; Al Hanifah et al., 2023; Uyen et al., 2021; Budi et al., 2020). Students are therefore required not only to understand mathematical concepts, but also to articulate them orally, symbolically, visually, and in written form. In this model, mathematical communication is positioned as an integral component of the learning process rather than as a final output.

With respect to visual representation, reflective problem based learning contributes significantly to students' ability to express mathematical ideas through diagrams, images, and graphical representations. The stages of problem exploration and investigative planning encourage learners to visualise mathematical concepts in a more concrete and meaningful manner. These findings are consistent with prior research highlighting the role of mathematical visualisation in clarifying conceptual relationships and supporting interactive learning processes (Aini et al., 2020; Svitek et al., 2022; Zeynivandnezhad et al., 2020; Herrera et al., 2024; Kohen et al., 2019; Rolfes et al., 2020). Moreover, the results reinforce previous evidence suggesting that visual representations facilitate the understanding of complex mathematical concepts and reduce excessive abstraction (Polotskaia & Nadon, 2021; Ruamba et al., 2025; Wang et al., 2020). The reflective dimension of the learning process enables students to reassess the accuracy and relevance of their visual representations, resulting in more precise and conceptually grounded outputs.

In relation to mathematical expression, students were trained to construct mathematical models through iterative reflection, collaborative discussion, and the formulation of alternative solutions. Consistent with existing literature, the ability to express mathematical ideas symbolically is recognised as a foundational element of mathematical communication (NCTM, 2020; Thinwiangthong et al., 2024; Budi et al., 2020;

Teledahl et al., 2024). The results of this study demonstrate that students engaged in reflective learning are better able to establish coherent connections between symbols, concepts, and contextual problem situations. These findings corroborate earlier research indicating that cognitive reflection supports students in articulating mathematical reasoning in a more systematic and critical manner (Maslukha & Ekawati, 2018; Saputra & Zulmaulida, 2021; Young & Shtulman, 2020; Calkins et al., 2019; Shtulman & Young, 2022; Dolapcioglu & Doğanay, 2020). Through reflection, students are able to identify errors in reasoning and refine inaccurate symbolic representations.

Students' mathematical reasoning in written form also showed marked improvement through reflective problem based learning. Learners were encouraged to synthesise mathematical ideas into coherent, logical, and argumentative written explanations. These findings are aligned with studies demonstrating that explicit instructional strategies within problem based learning can enhance mathematical reasoning and writing skills (Bundock et al., 2021; Chao et al., 2025; Irwandi et al., 2024; Riccomini et al., 2024). The results indicate that students became more proficient in constructing structured arguments, using appropriate mathematical terminology, and providing well supported justifications. Reflective problem based learning therefore contributes not only to conceptual understanding, but also to the overall quality of students' written mathematical communication.

Overall, the findings of this study are consistent with previous research reporting the effectiveness of problem based learning in fostering critical thinking and mathematical communication skills. The present study extends existing literature by explicitly integrating reflective components into each stage of the problem based learning process. Unlike conventional problem based learning models that often emphasise task completion, the reflective orientation prioritises self evaluation, solution justification, and knowledge reconstruction. These results reinforce earlier evidence suggesting that structured reflection is a critical factor in amplifying the impact of problem based learning on higher order thinking skills (Wan Husin et al., 2016; Maslukha et al., 2018; Lawal et al., 2020).

This study strengthens the conceptual framework of problem based learning by positioning reflection as a core element in the development of critical thinking and mathematical communication. The proposed model integrates principles of social constructivism, cognitive reflection, and andragogy

within a coherent instructional design. The findings have important implications for lecturers and mathematics educators, suggesting that reflective problem based learning may serve as a strategic alternative for improving learning quality in higher education. The model is particularly relevant for basic geometry courses and other mathematical contexts that require advanced critical thinking and communication skills. This study has several limitations that should be acknowledged. First, the sample was limited to students from a single study programme within one institution, which may restrict the generalisability of the findings to broader educational contexts. Second, the quantitative research design did not allow for an in-depth exploration of students' reflective thinking processes during the implementation of reflective problem-based learning. Future research is therefore recommended to involve more diverse institutional settings to enhance external validity. Additionally, adopting mixed-methods approaches would enable deeper investigation of students' cognitive and reflective processes. Further studies may also examine the impact of reflective problem-based learning on other relevant variables, such as mathematical disposition, metacognition, and mathematical creativity.

5. CONCLUSION

This study concludes that reflective problem based learning is significantly more effective than conventional instructional approaches in improving students' critical thinking and mathematical

communication skills. The findings confirm that the integration of authentic problem solving with structured mathematical reflection plays a crucial role in fostering higher order thinking skills and enhancing the quality of mathematical communication. Improvements in critical thinking are evidenced by students' increased capacity to analyse problems, evaluate solution strategies, and draw conclusions based on logical mathematical reasoning. At the same time, mathematical communication skills developed through enhanced visual representation, more accurate symbolic expression, and the production of systematic and coherent written arguments. The reflective process not only strengthened students' conceptual understanding but also improved the quality of their mathematical thinking and communication processes. This study contributes to the body of research on problem based learning by emphasising reflection as a central element that amplifies the effectiveness of PBL within mathematics education. From a practical perspective, the findings support the adoption of reflective problem based learning as a strategic and relevant pedagogical approach for mathematics instruction in higher education, particularly in developing competencies aligned with twenty first century learning demands. Nevertheless, this study is limited by its contextual scope and its predominantly quantitative design. Future research is therefore recommended to involve broader educational contexts, employ mixed methods approaches, and examine the influence of reflective problem based learning on additional affective and metacognitive variables.

Abbreviate

IQF: Indonesia Qualification Framework is the national qualification framework of Indonesia based on Presidential Decree No. 8/2012

HDI: The Human Development Index is a summary measure of average achievement in key dimensions of human development: a long and healthy life, being knowledgeable and having a decent standard of living. The HDI is the geometric mean of normalized indices for each of the three dimensions.

NA: National Assesment is the result of our evaluation on National Exam (UN)'s system and impact, coupled with adaptation and adjustment to international competency standards.

PISA: Programme for International Student Assessment is the OECD's Programme for International Student Assessment. PISA measures 15-year-olds' ability to use their reading, mathematics and science knowledge and skills to meet real-life challenges.

NCTM: National Council of Teachers of Mathematics is the world's largest mathematics education organization founded in 1920.

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